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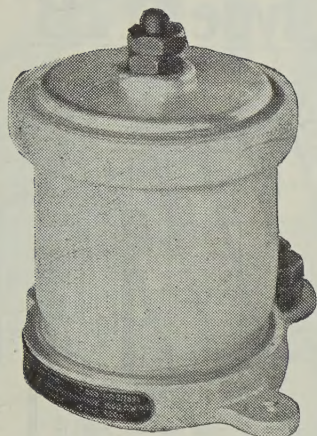
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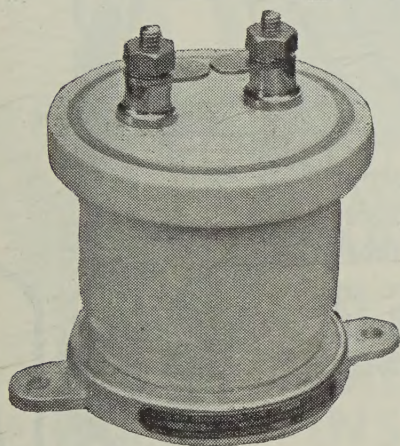
THE CONCEPT OF CAUSALITY
By PROF. MAX PLANCK

The seventeenth Guthrie Lecture, delivered on June 17, 1932.

I AM deeply sensible of the honour which you have conferred upon me in having allowed me to speak at the present meeting, which is devoted to the memory of Frederick Guthrie, one of the founders of the Physical Society of London. My pleasure is the greater in accepting your kind invitation, as I too rank amongst the physicists who have followed the steps of this celebrated investigator, inasmuch as his scientific career led him from University College, London, to German Universities, to Heidelberg and to Marburg.

It is true that a physicist of to-day, particularly if he is only a theorist, cannot dream of mastering the whole range of subject-matter which had to be understood as a matter of course by a physicist in the middle of last century. For Guthrie was not only a physicist, he was also a professional chemist; and it is just in the boundary province of these two disciplines that lies his most important work. His investigations on capillarity and heat-conduction have made him well known throughout the world, and his name will for all time be connected with the concepts of critical solution-temperature, of the eutectic alloy and of kryohydrate.

To-day the investigator who wishes to further the advance of his science substantially is obliged, by the rich abundance of accumulated material, to specialize narrowly the subject-matter of his researches. But on the other hand this involves the danger that he will find it more and more difficult to awaken any interest in his work among his scientific fellow-workers, engaged in different problems.

In order nevertheless to approach my task to-day with some hope of not boring you, I have chosen a subject which, though of rather an abstract nature, is just for that reason of fundamental significance for all domains of physics, and indeed for the whole of science—the question of the concept of causality. It is a question which could scarcely have been put forward a generation ago, when the classical theory was in its prime. But to-day, when the supporting pillars of the theory are in danger of tottering, this question stands in the centre of our interest.

Whoever seeks to clarify the opposing opinions in the present quarrel about the significance and the validity of the law of causality in modern physics, must naturally begin by stating that primarily everything depends on an agreement as to the sense in

which the word "causality" is used in physical science. At the outset we agree that in speaking of a causal link between two successive events we mean a certain connection, subject to law, between the two events, of which the earlier event is called the cause, the later one the effect. But the question is, in what does this particular kind of connection consist? Is there any infallible sign proving a certain event occurring in nature to be causally conditioned by another?

The numerous investigations hitherto undertaken concerning this question show that the surest approach to a clear answer is obtained by connecting the question with the possibility of making correct predictions of the future. Indeed, for proving that any two events are causally connected, there is no more unobjectionable means than that which consists in showing that from the occurrence of one event the occurrence of the other event can always be concluded in advance. That was already known to the farmer, who demonstrated *ad oculos* to the incredulous peasants the causal connection between artificial manure and fertility of soil. The peasants refused to believe that the lush growth of clover on the farmer's field was due to artificial manure, and sought for other reasons. So the farmer had certain narrow furrows ploughed on his field; then he shaped them into letters and manured them profusely, so that after the shooting up of the seed the following sentence was legible in distinct clover-writing: "This strip of land has been manured with gypsum."

As a starting-point for all further considerations I will therefore use the following simple proposition, applicable also beyond the domain of physics: *An event is causally conditioned if it can be predicted with certainty.* Thereby of course I only wish to say that the possibility of making a correct prediction for the future forms an infallible criterion for the existence of a causal connection, not by any means that the two mean one and the same thing. I need only recall the well-known example, that in the daytime we are quite able with certainty to predict the advent of night, yet day is not the cause of night.

But on the other hand we often assume the existence of a causal connection in cases where there is no possibility at all of a correct prediction. Think of the weather forecasts! The unreliability of weather prophets has become proverbial; and yet there is no trained meteorologist who does not look upon the occurrences in the atmosphere as causally determined. Thus we see that if we wish to reach the essence of the concept of causality, we must sift matters more minutely.

In the case of weather forecasts we may easily suppose that their unreliability is only conditioned by the size and the complicated nature of the object under consideration—the atmosphere. If we take only a small quantity of it, say a litre of air, we shall far more probably be able to make correct predictions as to its behaviour under external influences, such as compression, heat, moisture and the like. We know certain physical laws which enable us to predict more or less positively the results of the corresponding measurements, such as increase of pressure, increase of temperature, or condensation.

On closer observation, however, we arrive at a very remarkable conclusion. Even if we choose ever so simple conditions and use ever such delicate measuring instruments, we shall never succeed in calculating in advance the results of our

measurement with absolute accuracy; that is to say, not so accurately that it will agree with the measured number to the last decimal place. There always remains some residuum of inaccuracy, in contrast with purely mathematical calculations such as those of $\sqrt{2}$ and π which can be stated exactly to any number of decimal places. And what applies to mechanical and thermal phenomena is valid for all regions of physics, including electrical and optical phenomena.

Our experiences therefore compel us to recognize the following statement as a given and established fact: *In not a single instance is it possible to predict a physical event exactly.*

On placing this fact side by side with the proposition, which served as our starting-point, that an event is causally conditioned if it can be predicted with certainty, we are confronted by a vexatious but unavoidable dilemma. Either we stick to the wording of the original proposition, so that there is not a single instance in nature in which a causal connection can be asserted; or we make room for the assumption of a strict causality, when we are compelled to subject our original proposition to a certain modification.

There are nowadays a number of physicists and philosophers who have decided in favour of the first alternative; I will call them the indeterminists. According to them there is absolutely no real causality in nature—no strict law. It is only an illusion given us by the appearance of certain rules which are never absolutely valid, although often they are very approximately so. On principle the indeterminist seeks a root of a statistical kind for every physical law, for gravitation, and for electrical attraction. For him they are all laws of probability, only relating to mean values from numerous homogeneous observations, and possessing only approximate validity for single observations, always admitting, therefore, of exceptions.

A well-known example of such a statistical law is the dependence of gas-pressure on the density of gas and on temperature, in accordance with the kinetic theory of gases.

This theory is directly confirmed by investigating the fluctuations with time which occur in the pressure acting on a very small part of the sides of the vessel. It is well known that fluctuations of this kind, caused by the irregular impacts of the molecules, are quite universally observed, whenever high-speed molecules are in touch with bodies susceptible of motion. They are shown in the so-called Brownian molecular movement and also in the behaviour of an extremely sensitive balance, which never comes to complete rest but incessantly performs little irregular oscillations around its position of equilibrium.

And as in this case of the gas laws, the indeterminist wishes to attribute also all other physical laws ultimately to chance. For him nature is governed by statistics alone. His aim is to base physics on the calculus of probability.

But in fact the science of physics has hitherto developed on the opposite basis. It has chosen the second of the two alternatives: that is to say, in order to be able quite strictly to maintain the law of causality, it has slightly modified the starting-point, which was that an event was causally conditioned if it could be safely predicted. The modification consists in using the word "event" in a slightly altered

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sense. It is not to one single actual measurement, always containing casual and unessential elements, that the theoretical physicist gives the name of event. He reserves this name to an imagined process, going on in another world: we will call it the physicist's world-picture, which is substituted for the actual one given by our senses and by measuring instruments acting as a kind of refined sense. The physicist's world-picture is a mental construction, arbitrary to a certain extent; an idealization, created for the purpose of escaping from the uncertainty which inheres in every individual measurement, and of becoming able to establish sharply defined conceptual relations. In physics therefore all measurable quantities—lengths, intervals of time, masses, charges and the rest—have a double meaning, according as to whether we consider them as given directly by measurement, or as transferred into the physical world-picture. In the first meaning such quantities can only be defined inaccurately and can therefore never be represented by precise numbers. But in the physicist's world-picture they stand for definite mathematical symbols, which can be operated with according to strict rules. If in physics we make use of a trigonometric equation for calculating the height of a tower, then in speaking of the height we mean quite a definite thing, a well-defined quantity. Yet the actual measurement of the height does not furnish a definite quantity. Consequently the so-called true height of the tower is a different thing from the measured height. Exactly the same argument applies to the frequency of vibration of a pendulum or to the brightness of an incandescent lamp. Likewise every universal constant, for instance the charge on an electron, is a different thing from the actually measured charge.

The clear and logical distinction between the magnitudes and quantities of the world of sense and the similarly named magnitudes and quantities of the physicist's world-picture is absolutely indispensable for the clarifying of conceptions. Without this distinction a discussion about these questions is futile.

Therefore it is wrong to state (as some do) that the world-picture of physics contains or ought to contain only directly observable quantities. On the contrary, directly observable quantities do not appear at all in the world-picture. It contains nothing but symbols. Besides, there are always in the world-picture elements which for the world of the senses have only a very indirect significance or none at all, such as ether-waves, partial vibrations, coordinate systems and the like. Such elements at first act as ballast, but they are put up with in view of the decided advantage afforded by the introduction of the mental world-picture. This advantage is, that the world-picture enables us to carry through a strict determinism.

To be sure, the world-picture always remains an auxiliary conception: what we are eventually concerned with is of course the events in the world of the senses and their approximately correct forecasting, which in classical theory is effectuated in the following manner. First an object of the world of the senses, say a system of material bodies in any measured state, is symbolized, that is to say transferred into the world-picture. In this way a definite physical system in a definite initial state is obtained. In like manner the influences which are subsequently exerted upon the object from the outside are replaced by corresponding symbols in the framework of the world-picture. Thus we are provided with the external forces acting on the

systems, or with the boundary conditions. By these data the behaviour of the system is for all time unambiguously defined and can be calculated with absolute accuracy from the differential equations of the theory. Thus the coordinates and the momenta of all particles of the system result in quite definite functions of time. Now if for any later time we transfer back into the world of the sense the symbols used for the world-picture, we obtain a connection between a later event in the world of sense and an earlier event in the world of sense. This connection can then be utilized for the approximate prediction of the later event.

To summarize: while in the world of sense the prediction of an event is always affected by something of an uncertainty, in the physicist's world-picture all events follow certain definable laws; they are strictly determined causally. Therefore the introduction of the world-picture reduces the uncertainty in the prediction of an event of the world of sense, to the uncertainty of the translation of the event from the world of sense to the world-picture and vice versa. Herein lies the significance of the physicist's world-picture.

In classical theory, without much bothering about this uncertainty, attention was concentrated on the elaboration of the causal view of what is going on in the ideal world-picture. That is how it has achieved its great successes. In particular, it has succeeded in finding a satisfactory interpretation for the irregular fluctuations mentioned above, which correspond to the pressure of a gas or to the Brownian movement—an interpretation that was based on the assumption of strict causality. For the indeterminists no real problem existed here. As they seek irregularity behind every rule, statistical law is what immediately satisfies them. Therefore they content themselves with the assumption that the collision of two individual molecules, as well as the impact of the molecules on the sides of the vessel, occurs only according to statistical laws. However, there is as little conclusive reason for such an assumption as there is for assuming that, because the electrons gather on the surface in a charged conductor, the charge of an individual electron must be on its surface too. On the other hand, the determinists, who conversely seek a rule behind every irregularity, were led to the task of building up a theory of the gas laws on the assumption that the collision of two individual molecules is strictly conditioned causally. The achievement of this task is the life-work of Ludwig Boltzmann. It forms one of the greatest triumphs of theoretical research. For Boltzmann's theory leads to the statement—confirmed by measurements—that the average energy of the fluctuations around the position of equilibrium is proportional to the absolute temperature. And further: from the measurement of such oscillations, for instance those of an extremely sensitive torsion-balance, this theory makes possible a remarkably accurate calculation of the absolute number and mass of the striking molecules.

In view of these and other great successes, reasonable hope prevailed that the world-picture of classical physics would on the whole be equal to its task, and that the uncertainties remaining after the transfer into and from the world of the senses would lose their importance as experimental methods improved in refinement. But with one stroke this hope has for all time been destroyed by the appearance of the elementary quantum of action.

The so-called principle of uncertainty, discovered and formulated by Heisenberg, is characteristic of quantum physics. It states that of two canonically conjugated quantities, such as position and momentum, or time and energy, only one can be measured with absolute accuracy, and that only by the sacrifice of accuracy in the other. That is to say that by increasing the accuracy with which one of them is measured you diminish the accuracy of the other, the product of the two errors being constant. Hence, if one of the two is determined with absolute accuracy, the other remains absolutely undetermined.

It stands to reason that this statement makes it on principle impossible to transfer with any accuracy into the world of the senses the simultaneous values of coordinates and momenta which play the predominant part in the world of classical physics. For the strictly causal view of the world this fact raises a difficulty, which has already led some indeterminists to affirm that the law of causality in physics is definitely disproved. However, on closer consideration this conclusion, which is due to confusion of the world-picture with the world of sense, must be called at least premature. For there is at hand, for overcoming this difficulty, a means which has often done excellent service in similar cases. It is the assumption that the question as to the simultaneous values of the coordinates and of the momenta of a particle has no meaning in physics. The law of causality must not be blamed for the impossibility of answering a meaningless question. The blame must rather be laid on the assumptions which have led to the putting of that question, that is to say on the assumed structure of the physicist's world-picture. And as the classical world-picture has failed, it must be replaced by another.

In fact this has been done. The new world-picture of quantum physics has arisen from the desire to render possible the accomplishment of a strict determinism in spite of the existence of the quantum of action. For this end the material particle, which had hitherto formed the primary component of the world-picture, has had to be divested of its elementary character: it has been dissolved into a system of material waves, which form the elements of the new mental picture of the world. The particle in its old meaning now forms only a special limiting case as an infinitely narrow wave-group, of which—following Heisenberg's principle of uncertainty—the momentum in a specified position of the particle is quite indefinite. If we allow a certain latitude to the position of the particle, the momentum will also attain an approximately definable value. The laws of classical mechanics are then approximately valid for both position and momentum.

In general the laws of wave mechanics are, as everybody knows, quite different from those applying to particles in classical mechanics. But the most important point is that the function which is characteristic for the material waves, the wave function or the probability function (the name is irrelevant here), is completely determined for all places and times by the initial conditions and the boundary conditions. We can calculate it by quite definite rules, employing either Schrödinger's operators or Heisenberg's matrices or Dirac's q-numbers.

Thus we see that the world-picture in quantum physics is governed by the same rigorous determinism which rules classical physics. It is only that the symbols are

different, and that we operate with other rules of calculation. Accordingly in quantum physics, as formerly in classical physics, the uncertainty in the prediction of events of the world of sense is reduced to the uncertainty of the connection between the world-picture and the world of sense; that is to say, to the uncertainty of the translation of the symbols of the world-picture into the world of sense and vice versa. The fact that this double uncertainty is put up with forms the most impressive proof of the importance of the task of maintaining determinism in the world-picture.

Yet but a superficial glance enables us to recognize how very far in quantum physics the world-picture has diverged from the world of the senses, and how much more difficult it is to transfer an event from the world-picture to the world of the senses, or vice versa, in quantum physics than it formerly was in classical physics. In classical physics the meaning of every symbol was immediately comprehensible: the position, the velocity, the momentum, the energy of a particle could be stated more or less directly from measurements. There was no evident reason for not assuming that one should be able to reduce the remaining uncertainty below any limit, as the refinement of experimental methods progressed. On the other hand, in quantum mechanics the wave function yields no means whatever whereby this function can be interpreted directly in the world of sense, the first obstacle being that it does not refer to ordinary space but to configuration space, which possesses as many dimensions as there are independent coordinates in the physical system under discussion. Furthermore—and this is the important point—the wave function does not furnish the values of the coordinates as functions of time; it only furnishes the probability for the coordinates possessing any given values at a definite given time.

This circumstance has again incited the indeterminists to an attack upon the law of causality. And this time the attack seems to promise a positive success; for from all measurements nothing more than a statistical significance of the wave function can be deduced. But again the same loop-hole for escape is open to champions of strict causality. They assume that the question as to the significance of a definite symbol in the world of quantum physics—for instance a material wave—has no definite meaning, as long as we are not at the same time told how to ascertain this meaning—not told in what condition is the special instrument which is employed for transferring the symbol into the world of sense. We therefore also speak of a causal effect of this instrument. Thereby we imply that the inaccuracy under discussion is at least in part conditioned by the fact that the amount of the quantity to be measured depends in a certain manner, subject to law, on the nature of the measuring-process.

With this auxiliary assumption the whole question has been led into channels the further course of which still remains dark. For now the indeterminists are justified in putting forward the question whether any sensible meaning can be attributed to the idea that the measuring-instrument should exert a causal influence on the process to be measured; for any attempt to test this influence would require new measurements, which would involve a new causal interference and would therefore bring a new feature of uncertainty into the problem.

And yet this objection does not finish the matter. For, as every experimental

physicist knows, there are not only direct but also indirect testing methods. In many cases the latter have done good service where the former had failed. Above all I wish to oppose the now widespread and seemingly plausible opinion, that a question in physics is only worth investigation if from the outset the fact that it admits of a definite answer is established. If the physicists had always followed this precept, the celebrated experiment of Michelson and Morley on the measurement of the so-called absolute velocity of the earth would never have been made. We should then perhaps not even to-day be in possession of the theory of relativity. So our efforts to ascertain the absolute velocity of the earth have proved exceedingly fruitful for science, although nowadays the question itself is almost universally considered to be meaningless. Then are we not justified in expecting even much greater profit from investigating the problem of causality, the roots of which have certainly not been reached hitherto—a problem quite pre-eminent in its fertilizing influence on research?

But how to come to a decision? Evidently there is nothing for it but to take one's choice between the opposing standpoints, to adopt one, and then to see whether from this starting-point we attain valuable or useless results. In this respect we must welcome the fact that the physicists who are interested in this subject are divided into two camps, one inclining to determinism, the other to indeterminism. As far as I see, the latter are at present in the majority. But it is hard to tell and the situation may easily change in the course of time. In between there seems to be room for a third party, occupying an intermediate position. They attribute to certain concepts, such as electrical attraction and gravitation, an immediate significance and a strict rule of law, while ascribing to other concepts, such as the light wave and the particle wave, only a statistical significance for the world of sense. This notion however appears at the outset rather unsatisfactory on account of its want of uniformity. So I shall leave it aside and confine myself to the elucidation of the two absolutely logical standpoints.

The indeterminists' yearning after knowledge is satisfied by the statement that the wave function of quantum physics is only a probability scheme; for him there is no further question to put. On the other hand he looks upon a definite law of nature, such as Coulomb's law of electrical attraction, as an unsolved problem. He cannot content himself with Coulomb's law of force or potential, but must try to find exceptions. He will not be satisfied unless he succeeds in establishing what the probability is that the electrical force will diverge from Coulomb's law to any specified intent.

The determinist takes the opposite view in all these matters. He is quite content to look upon Coulomb's law as an ultimate and fundamental law of nature. The interpretation of the wave function as a probability function he will only admit as long as no account is taken of the particular apparatus with which the wave is generated or analysed. He seeks relations strictly subject to law between what is going on in the bodies that interact with the wave, and the form of the wave function. For this purpose he is of course obliged to begin by making the measuring-apparatus as well as the wave function the object of his research. That is to say, he must

transfer into his world-picture the whole experimental arrangement for generating the material waves (for instance the high-potential battery, incandescent wire, or radio-active substance) and also the whole of the measuring-apparatus such as the photographic plate, ionization chamber, or Geiger counter, with all that is going on in them; he must treat all these objects together as one single system, as a closed unity. This, of course, is not sufficient to solve the problem, which on the contrary has become even more complicated. For since one is allowed neither to divide the total system nor to expose it to any interference from outside, lest it should lose its original character, no direct test whatever is feasible. On the other hand it now becomes possible to set up certain hypotheses of a new kind with regard to the internal occurrences, and then to examine their consequences. The future will show whether we are able to advance in this way; up to now we cannot distinctly discern in what direction progress will be accomplished. But this much may be safely affirmed: the elementary quantum of action sets an objective insuperable limit to the sensitiveness of the physical measuring-apparatus at our disposal, and the progress we strive for will consequently exhibit this limit more plainly than heretofore.

This really brings us to the end of our considerations. They have shown us that the standpoint even of modern physics does not prevent us from achieving a strictly causal view—the word "causal" being understood in the modified sense explained above—although the necessity for such a view can be proved neither from the outset nor afterwards. Yet even the convinced determinist—and he perhaps more than anybody else—is overcome by a scruple which hinders him from being quite satisfied with the interpretation of causality introduced here. For even if we should succeed in further developing the concept of causality on the lines laid down, it would be affected by a grave deficiency. One might suppose that a relation of so deep a significance as the causal connection between two successive events would in its essence be independent of the human mind which considers it. The reverse is true. At the outset we had to attach the concept of causality to the human intellect, with reference to the capability of predicting an event; furthermore, we were not able to enforce the adoption of the deterministic view otherwise than by substituting for the given world of our senses the physical world-picture. The latter is a creation of human imagination of a provisional and changeable character. Anthropomorphisms of that sort are ill suited to form a fundamental physical concept. So the question arises whether there is no way of giving the concept of causality a deeper significance, by divesting it as much as possible of its anthropomorphic character and by making it independent of the introduction of an artefact, such as the physicist's world-picture. We must of course retain our original proposition, that an event is causally conditioned if it can be safely predicted; otherwise we shall lose our only foothold. But we must also adhere to the second proposition, that in not a single case is it possible to predict an event exactly. It then follows, as before, that in order to be able to speak of causality in nature we must apply some modification to the first statement. So far everything remains as it was. But the modification we had applied above can be replaced by one of quite a different kind, in one sense quite an opposite modification.

What we modified there was the object of prediction, the event. We did not refer the events to the immediately given world of the senses, but to the fictitious world-picture. Thus we were able accurately to determine the events. But instead of the object we may modify the subject of prediction, the predicting mind. For every prediction necessitates the existence of a predictor. In the following we shall therefore limit our attention to the predicting subject, and look upon the immediately given events of the world of sense as the object of prediction, without introducing an artificial world-picture.

It is obvious that the certainty of the prediction depends in a high degree upon the individuality of the predictor. Let us again refer to weather forecasts: it is evident that it makes a great difference who provides us with the forecast for to-morrow—whether it is an ignorant person, who knows nothing about to-day's atmospheric pressure, direction of the wind, atmospheric temperature and humidity, or a practical farmer, who has noted all these data and has a wide experience, or again a scientifically trained meteorologist, who besides the local data has at his disposal numerous weather-charts from all parts with exact information. With each of these successive prophets the uncertainty of the prediction is more and more diminished. It is therefore an obvious thought to assume that an ideal mind, apprehending everywhere all the physical occurrences of to-day in their minutest points, should be able to predict with absolute accuracy the weather of to-morrow in all its details. And the same argument can be applied to every other prediction of physical events.

Such an assumption means an extrapolation, a generalization, which can neither be maintained by a logical conclusion nor refuted *a priori*. It must therefore not be judged according to its truth but rather according to the value that is inherent in it. In the light of this view, the actual impossibility of accurately predicting an event in even one single case, either from the standpoint of classical physics or from that of quantum physics, is a natural consequence of the circumstance that man with his senses and his measuring-instruments is a part of nature. He is subject to her laws and cannot escape from her, while such a tie does not exist for the ideal mind. The objection that this ideal mind is only a product of our thoughts, and that our thinking brain also consists of atoms following physical laws, is not able to withstand a closer test. For it is indubitable that our thoughts can lead us beyond every law of nature known to us, and that we are able to imagine relations which transcend the realm of physics.

It is true that in order to accomplish such a notion, logically we must subject ourselves to a severe restriction: we must forego making the ideal mind the object of a scientific investigation. We must not consider the ideal mind akin to us, and must not demand of it how it procures the knowledge enabling the exact prediction of future events. For the inquisitive questioner might easily, like Faust, be awed by the answer: "Thou'rt like the spirit which thou comprehendest, not me!" And if the questioner nevertheless remains obdurate and declares that the notion of an ideal mind, if not illogical, is yet empty and superfluous, he must be met with these arguments: Not all statements eluding logical reasoning are scientifically valueless,

and such a short-sighted formalism chokes up the source at which men like Galilei, Kepler, Newton and many other great physicists have slaked their scientific thirst for knowledge. For all these men, consciously or unconsciously, the devotion to science was a matter of faith, of unwavering faith in a rational scheme of the universe.

It is true, this faith can be forced upon nobody, just as one cannot command truth or forbid error. But the simple fact that up to a certain degree we are able to subject future natural events to our thoughts and to guide them at our will would remain a complete riddle, if it did not at least point to the existence of a certain harmony between the outer world and the human mind. And the question to what depths one imagines the sphere of this harmony to be extended is only of secondary importance.

In conclusion we may therefore say: the law of causality is neither right nor wrong, it can be neither generally proved nor generally disproved. It is rather a heuristic principle, a sign-post (and to my mind the most valuable sign-post we possess) to guide us in the motley confusion of events and to show us the direction in which scientific research must advance in order to attain fruitful results. As the law of causality immediately seizes the awakening soul of the child and causes him indefatigably to ask "Why?" so it accompanies the investigator through his whole life and incessantly sets him new problems. For science does not mean contemplative rest in possession of sure knowledge, it means untiring work and steadily advancing development.

THE AXIAL SOUND-PRESSURE DUE TO DIAPHRAGMS WITH NODAL LINES

By N. W. McLACHLAN, D.Sc., M.I.E.E.

ABSTRACT. A formula is found for the axial sound-pressure due to a disc having a nodal circle and vibrating in an infinite rigid plane. Beyond a certain axial distance, when the nodal circle occurs at $r = a/\sqrt{2}$, the pressure vanishes owing to interference caused by the inner and outer portions of the disc vibrating in opposite phase. The case of n nodal circles of arbitrary radii is treated by an approximate method. A rigid disc is imagined to be severed around each nodal circle, whilst contiguous annuli vibrate with equal amplitudes in opposite phases. Finally, the pressure on the axis of a conical shell having nodal circles is treated as in the previous case. When the semi-apical angle of the cone is $\frac{1}{2}\pi$ and there are no nodal circles, the formula reduces to that for a rigid disc.

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§ 1. FREE-EDGE DISC WITH NODAL CIRCLE

p, r_1, s THE sound-pressure dp at a point P distant r_1 from an elementary area ds vibrating in an infinite rigid plane as shown in figure 1, is given by*

$$dp = \frac{\rho}{2\pi} \frac{e^{-ikr_1}}{r_1} \frac{\partial^2 w}{\partial t^2} ds \qquad \ldots\ldots(1),$$

ρ, w where ρ is the density of the medium, $\partial^2 w/\partial t^2$ the acceleration of the element
ω, c normal to the plane, $k = \omega/c$, c being the velocity of sound, while $\omega = 2\pi \times$ frequency.

Let the dynamic deformation curve of the disc be of the form

$$w = A\,(1 - 2r^2/a^2) \qquad \ldots\ldots(2),$$

A, X where
$$A = X \cos \omega t.$$

r, R_1 From figure 1
$$r^2 = r_1{}^2 - R_1{}^2 \qquad \ldots\ldots(3).$$

Substituting for r^2 from (3) in (2) we get

$$w = A\,\{1 - 2\,(r_1{}^2 - R_1{}^2)/a^2\}$$

and, therefore, the axial acceleration†

$$\frac{\partial^2 w}{\partial t^2} = \ddot{A}\,\{1 - 2\,(r_1{}^2 - R_1{}^2)/a^2\} \qquad \ldots\ldots(4),$$

where
$$\ddot{A} = -\,\omega^2 X \cos \omega t.$$

Differentiating (3) we have

$$2r\,dr = 2r_1\,dr_1,$$

so that
$$ds = 2\pi r\,dr = 2\pi r_1\,dr_1 \qquad \ldots\ldots(5),$$

R_2 the limits of r_1 being R_2 and R_1.

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* Rayleigh, *Sound*, 2, 162 (1894), equation (1).
† The acceleration of any point on the disc, parallel to the axis.

Substituting from (4) and (5) in (1), the axial pressure at P due to the whole disc is, therefore, given by

$$p = \rho \ddot{A} \int_{R_1}^{R_2} e^{-ikr_1} \left\{ 1 - \frac{2(r_1^2 - R_1^2)}{a^2} \right\} dr_1 \qquad \ldots\ldots(6).$$

Performing the integration in (6), inserting the limits, and arranging terms, we ultimately find that

$$p = \frac{2\rho \ddot{A} e^{-i\alpha}}{k} \left[\cos \beta - \frac{2}{ka^2} (R_2 + R_1) \sin \beta + i \left\{ \frac{4}{k^2 a^2} \sin \beta - \frac{2}{ka^2} (R_2 - R_1) \cos \beta \right\} \right]$$

$$\ldots\ldots(7),$$

where
$$\alpha = \tfrac{1}{2} k (R_2 + R_1),$$

and
$$\beta = \tfrac{1}{2} k (R_2 - R_1).$$
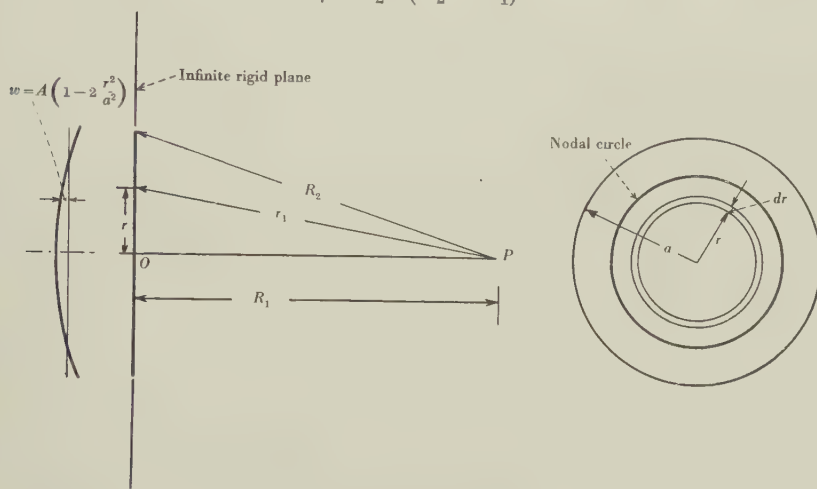
Fig. 1. Diagram illustrating free-edge disc vibrating with one nodal circle, and set in an infinite rigid plane. The dynamic deformation curve is $w = A(1 - 2r^2/a^2)$.

At a great distance when $R_1 \gtrless a$ we can write

$$R_2 + R_1 = 2R_1, \quad R_2 - R_1 = a^2/2R_1,$$

so that
$$\alpha = kR_1 \text{ and } \beta = ka^2/4R_1.$$

Since β is very small $\cos \beta \doteqdot 1$ and $\sin \beta \doteqdot ka^2/4R_1$.

On insertion of these values in (7) the value of p is zero. This is due to the interference arising from the inner and outer portions of the disc vibrating in opposition. In equation (2) we tacitly assumed the nodal circle to occur where $r = a/\sqrt{2}$. If some other value were chosen, the pressure would have a value different from zero.

The modulus of p found in the usual way is

$$|p| = \frac{2\rho \ddot{A}}{k} \left[\frac{4}{k^2 a^4} \{ (a^2 + 2R_1^2) - 2R_1 \sqrt{a^2 + R_1^2} \cos 2\beta \} + \cos^2 \beta + \frac{16 \sin^2 \beta}{k^4 a^4} \right.$$

$$\left. - \frac{2 \sin 2\beta}{ka^2} \left\{ (\sqrt{a^2 + R_1^2} + R_1) + \frac{4}{k^2 a^2} (\sqrt{a^2 + R_1^2} - R_1) \right\} \right]^{\frac{1}{2}} \quad \ldots\ldots(8).$$

§ 2. FREE-EDGE DISC WITH n NODAL CIRCLES

In this case we shall use an approximate artifice to simplify the analytical treatment. Accordingly we take a rigid disc having n concentric nodal circles of arbitrary radii. The disc is assumed to be severed at each circle and contiguous annuli vibrate in opposite phase. The amplitude* of each annulus is represented by w, where $w = A = X \cos \omega t$.

w

From expressions (1) and (6) we have for any particular annulus

p_μ

$$p_\mu \doteq \rho \ddot{A} \int_{R_{\mu-1}}^{R_\mu} e^{-ikr_1} dr_1 \qquad \ldots\ldots(9),$$

R_μ

where R_μ is the distance from P to the outer edge of the annulus, and $R_{\mu-1}$ the distance from P to the inner edge of the annulus.

Remembering that consecutive annuli have opposite phases, the resultant pressure at P due to all the annuli is given by

$$p = \rho \ddot{A} \left[\int_{R_n}^{R_{n+1}} e^{-ikr_1} dr_1 - \int_{R_{n-1}}^{R_n} e^{-ikr_1} dr_1 + \ldots \right] \qquad \ldots\ldots(10)$$

$$= 2\rho \ddot{A} \{ e^{-i\alpha_n} \sin \beta_n + \ldots (-1)^n e^{-i\alpha_0} \sin \beta_0 \} \qquad \ldots\ldots(11),$$

α_n

where

$$\alpha_n = \tfrac{1}{2} k (R_{n+1} + R_n),$$

β_n

$$\beta_n = \tfrac{1}{2} k (R_{n+1} - R_n).$$

$R_n \ldots R_1$ are the distances of the annuli from P, and R_{n+1}, R_0 are the distances from P of the outer edge and centre respectively.

By converting the exponentials into circular functions the modulus of p can be found in the usual way. In the limit when $n \to \infty$ and the annuli are of equal radial width, the axial pressure $\to 0$. This is obvious since the axial pressures due to contiguous annuli are equal and opposite.

When there is only one nodal circle, formula (11) reduces to

$$p = 2\rho \ddot{A} \{ e^{-i\alpha_1} \sin \beta_1 - e^{-i\alpha_0} \sin \beta_0 \} \qquad \ldots\ldots(12).$$

Taking the radius of the nodal circle as $r = a/\sqrt{2}$, then at a great axial distance where $R_1 \gtrless a$ we have, from the geometry of figure 1, $\alpha_0 = \alpha_1$ and $\beta_0 = \beta_1$. Thus the pressure vanishes as in the preceding section, where a different procedure was used. By varying the radius of the nodal circle its influence on the axial pressure can be determined.

§ 3. CONICAL SHELL WITH NODAL CIRCLE

As in the previous section we shall adopt the artifice of a rigid shell severed at the nodal circle, the inner and outer portions vibrating in opposition with equal amplitudes.

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* Cases can be treated when the amplitudes of the annuli are different; the phases can also be suitably chosen to fit special conditions.

From the geometry of figure 2 when $R \gtrless a$ with adequate approximation we can write

$$r_1 = R - r \cot \tfrac{1}{2}\psi \qquad \qquad ......(13),$$

ψ being the plane apical angle of the shell.

From expression (1) the pressure at P due to an elementary ring of radius r is

$$dp = \rho \ddot{A} r_1^{-1} e^{-ikr_1} r \, dr \qquad \qquad ......(14).$$

So far as the amplitude of the sound-pressure is concerned the distance r_1 can be assumed constant and equal to R over the surface of the shell. The phase being paramount, since it controls the interference, it is essential to use the value of r_1 from (13) in the exponential index of (14).
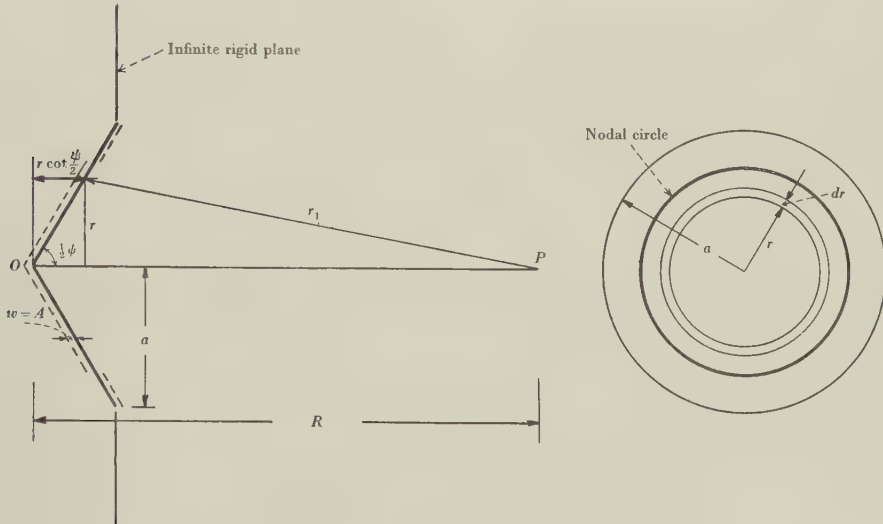
Fig. 2. Diagram illustrating conical shell vibrating with one (hypothetical) nodal circle, in an infinite rigid plane. The shell is severed at the nodal line; the inner and outer portions vibrate in opposite phase.

Thus (14) becomes

$$dp = \rho \ddot{A} R^{-1} e^{-ikR} e^{ik_1 r} r \, dr \qquad \qquad ......(15),$$

where $k_1 = k \cot \tfrac{1}{2}\psi$.

If the nodal radius is b the axial pressure at P due to the inner portion of the shell is p_1, where

$$p_1 = \frac{\rho \ddot{A}}{R} \int_0^b e^{ik_1 r} r \, dr \qquad \qquad ......(16),$$

the factor e^{-ikR} being omitted since it merely affects the phase.

The axial pressure at P due to the outer ring of the shell is

$$p_2 = \frac{\rho \ddot{A}}{R} \int_b^a e^{ik_1 r} r \, dr \qquad \qquad ......(17).$$

Moreover, the resultant pressure due to the whole shell is the difference between (17) and (16). Expanding $e^{ik_1 r}$ in circular functions we get for (17) and (16),

$$p = \frac{\rho \ddot{A}}{R} \left\{ \int_b^a r \cos k_1 r \, dr - \int_0^b r \cos k_1 r \, dr + i \left[\int_b^a r \sin k_1 r \, dr - \int_0^b r \sin k_1 r \, dr \right] \right\}.$$
$$......(18).$$

Integrating (18) and inserting the various limits we ultimately arrive at the formula*

$$p = \frac{\rho \ddot{A}}{R} \left\{ \frac{1}{k_1} \left(a \sin k_1 a - 2b \sin k_1 b \right) + \frac{1}{k_1^2} \left(\cos k_1 a - 2 \cos k_1 b + 1 \right) \right.$$

$$\left. - i \left[\frac{1}{k_1} \left(a \cos k_1 a - 2b \cos k_1 b \right) - \frac{1}{k_1^2} \left(\sin k_1 a - 2 \sin k_1 b \right) \right] \right\} \quad \ldots\ldots(19)$$

$$= \frac{\rho \ddot{A}}{R} \{ A_1 + iB_1 \}.$$

If we put $b = 0$, equation (19) reduces to that for a rigid conical shell vibrating as a whole. Under this condition when $\psi \to \pi/2$, $k_1 a$ is very small and we can write $\sin k_1 a = k_1 a$ and $\cos k_1 a = 1 - k_1^2 a^2/2$. Consequently when $b = 0$ and $\psi \to \pi/2$ expression (19) reduces to

$$p = \rho \ddot{A} R^{-1} \left(\tfrac{1}{2} a^2 + \tfrac{1}{2} i k_1 a^3 \right) \qquad \ldots\ldots(20).$$

Ultimately when $k_1 = 0$ the cone becomes a rigid disc of radius a and (20) can be written

$$p = \rho \ddot{A} a^2 / 2R \qquad \ldots\ldots(21),$$

which agrees with a well-known result given previously†. This checks the accuracy of (19).

The modulus of p in (19) is

$$|p| = \rho \ddot{A} R^{-1} \sqrt{(A_1^2 + B_1^2)}$$

$$= \frac{\rho \ddot{A}}{R k_1^2} \sqrt{\{ z_1^2 + 4 z_2^2 + 6 - 4 \left(z_1 z_2 + 1 \right) \cos \left(z_1 - z_2 \right) - 4 \left(z_1 - z_2 \right) \sin \left(z_1 - z_2 \right) }$$

$$+ 2 z_1 \sin z_1 - 4 z_2 \sin z_2 + 2 \cos z_1 - 4 \cos z_2 \} \qquad \ldots\ldots(22),$$

z_1 where $z_1 = k_1 a,$

z_2 $z_2 = k_1 b.$

The case of a rigid conical shell is obtained when $b = 0$; i.e. when $z_2 = 0$. Then expression (22) reduces to

$$p = \frac{\rho \ddot{A}}{R k_1^2} \sqrt{\{ z_1^2 + 2 - 2 \left(\cos z_1 + z_1 \sin z_1 \right) \}} \qquad \ldots\ldots(23),$$

which is identical with that found by Stenzel‡. In this case the axial pressure is not zero. It decreases slowly with increase in frequency, i.e. increase in k_1 or

$$\omega c^{-1} \cot \tfrac{1}{2} \psi,$$

c where c is the velocity of sound.

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* In obtaining the various formulae in this section, the influence of the air column vibrations within the shell and horn action of the shell in response to its own radiation at higher frequencies have been neglected.

† *Wireless World*, March 30, 1927; *Proc. R. S. A*, **122**, 606 (1929), equation (6).

‡ *Z. f. Tech. Phys.* **10**, 569 (1929).

When there are n nodal circles of radii a_1, a_2, ... a_n it is easy to show that

$$A_1 = \frac{1}{k_1} \{a \sin k_1 a - 2a_n \sin k_1 a_n + 2a_{n-1} \sin k_1 a_{n-1} - ...\}$$

$$+ \frac{1}{k_1^2} \{\cos k_1 a - 2 \cos k_1 a_n + 2 \cos k_1 a_{n-1} - ...\} \qquad ......(24),$$

$$B_1 = - \frac{1}{k_1} \{a \cos k_1 a - 2a_n \cos k_1 a_n + ...\}$$

$$+ \frac{1}{k_1^2} \{\sin k_1 a - 2 \sin k_1 a_n + ...\} \qquad ......(25),$$

where a is the outer radius of the shell.

If the transmission loss* in the shell is small, so that no phase changes in ampli-tude occur excepting at a nodal circle, where the phase is reversed, the dynamic deformation curve can be measured. Then if this is given by $f(r)$ the axial pressure becomes

$$\frac{\rho \ddot{A}}{R} \int_0^a e^{ik_1 r} f(r) \, r \, dr \qquad ......(26).$$

In general the integration will have to be performed graphically. Assuming that with one nodal circle $f(r)$ is parabolic, the integral can easily be evaluated.

* See letter to *The Wireless Engineer*, October, 1931.

THE ACCESSION TO INERTIA OF FLEXIBLE DISCS VIBRATING IN A FLUID

By N. W. McLACHLAN, D.Sc., M.I.E.E.

ABSTRACT. In this paper formulae are obtained for the velocity-potential at the surface of a free-edge disc vibrating with nodal lines in a fluid. These formulae are used to ascertain the accession to inertia due to the fluid when the disc is set in an infinite rigid plane. The equivalent mass and the mass coefficient of the disc vibrating *in vacuo* are found also. By means of these results, the influence of the fluid on the frequency of vibration with (*a*) one nodal circle, (*b*) one nodal diameter, (*c*) stationary centre, is evaluated. In air the alteration in frequency is almost negligible, whereas in water the frequency is reduced to a small fraction of its value *in vacuo*.

§ 1. INTRODUCTION

THE additional inertia of a body when it moves in a fluid is very familiar in hydrodynamics. Treatment of the problem from an acoustical and, therefore, vibrational viewpoint, has been confined mainly to rigid bodies, although H. Lamb* has examined the case of a clamped-edge flexible disc vibrating in its gravest mode in water†.

In the present paper it is proposed to deal with the accession to inertia of free-edge flexible discs having nodal lines. Although vibration in air is of primary interest, the analysis also holds for water or any other fluid. Computations will be made to show the enormous difference between the vibrational frequencies in air and in water.

The analysis is based on the following assumptions:

(1) The ratio of the radius of the disc to the wave-length is such that the distribution of radiation is substantially spherical.

(2) The disc vibrates in an infinite rigid plane situated in free fluid.

(3) The amplitude of the disc at any radius varies sinusoidally with time.

(4) Transmission loss in the disc is neglected.

(5) The shape of the disc is unaffected by fluid pressure, and true nodal lines —not merely positions of minimum amplitude‡—exist.

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* *Proc. R. S. A*, **98**, 205 (1920).

† This has been treated experimentally by J. H. Powell and J. H. T. Roberts, *Proc. Phys. Soc.* **35**, 170 (1923).

‡ *Phil. Mag.* **12**, 771 (1931).

§ 2. FREE-EDGE DISC WITH ONE NODAL CIRCLE

Let the dynamic deformation curve be of the type

$$w = A\left(1 - p_1 r^2/a^2\right), \qquad\qquad w$$

then

$$\partial w/\partial t = \dot{A}\left(1 - p_1 r^2/a^2\right),$$

where

$$A = X\cos\omega t, \qquad\qquad A, X, \omega$$

$$\dot{A} = -\,\omega X\sin\omega t;$$

a is the radius of disc; a

p_1 a parameter governing the radius of the nodal circle; p_1

and r is the distance of any point on the disc from the centre. r
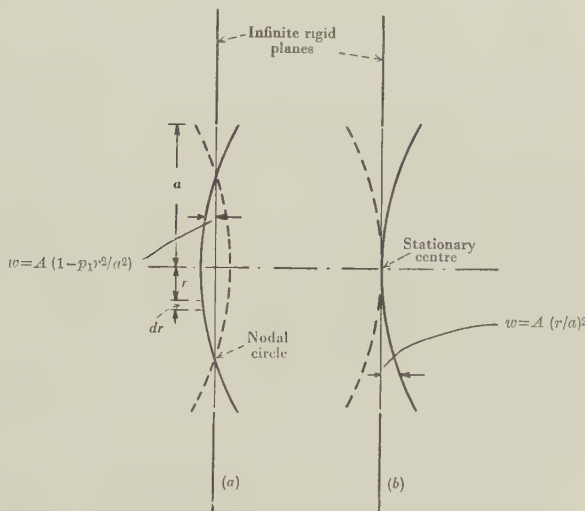
Fig. 1. Diagram illustrating free-edge disc vibrating in an infinite rigid plane (a) with one nodal circle; (b) with stationary centre. The dynamic deformation curve for (a) is $w = A\left(1 - p_1 r^2/a^2\right)$, and for (b) $w = A\,(r/a)^2$.

The velocity-potential ϕ at points on the disc is given* by ϕ

$$\phi = \int_0^\infty J_0\left(kr\right)dk \int_0^a J_0\left(kr\right)\frac{\partial w}{\partial t}\,r\,dr \qquad\qquad \ldots\ldots(1),$$

where

$$k = \omega/c,$$

$$\omega = 2\pi \text{ frequency,}$$

$$c = \text{velocity of longitudinal waves in fluid.}$$

This formula is applicable only when the deformed shape of the disc is symmetrical about its axis, e.g. it cannot be used for nodal diameters. Inserting the value of $\partial w/\partial t$ in (1) we obtain

$$\phi = \dot{A}\int_0^\infty J_0\left(kr\right)dk \int_0^a J_0\left(kr\right)\left(1 - p_1\frac{r^2}{a^2}\right)r\,dr \qquad\qquad \ldots\ldots(2).$$

Taking the first integral in (2) we find after a partial integration that

$$\int_0^a J_0\left(kr\right)\left(1 - p_1\frac{r^2}{a^2}\right)r\,dr = a^2\left\{(1 - p_1)\frac{J_1\left(ka\right)}{ka} + 2p_1\frac{J_2\left(ka\right)}{(ka)^2}\right\} \qquad \ldots\ldots(3).$$

* H. Lamb, *Hydrodynamics*, p. 129 (1906).

To complete the integration of (2) we require a formula pertaining to infinite integrals*, namely,

$$\int_0^\infty \frac{J_0(kr) J_n(ka)}{(ka)^n} \, dk = \frac{\pi^{\frac{1}{2}}}{2^n \, \Gamma(n + \frac{1}{2})} \frac{1}{a} F\left[-n + \frac{1}{2}, \frac{1}{2}, 1, \frac{r^2}{a^2}\right] \quad \dots\dots(4).$$

Putting $\left.\begin{array}{l} n = 1 \\ n = 2 \end{array}\right\}$ in (4) we obtain

$$\int_0^\infty \frac{J_0(kr) J_1(ka)}{ka} \, dk = \frac{1}{a} F\left(-\frac{1}{2}, \frac{1}{2}, 1, \frac{r^2}{a^2}\right) \qquad \dots\dots(5),$$

and

$$\int_0^\infty \frac{J_0(kr) J_2(ka)}{(ka)^2} \, dk = \frac{1}{3a} F\left(-\frac{3}{2}, \frac{1}{2}, 1, \frac{r^2}{a^2}\right) \qquad \dots\dots(6).$$

Introducing the factors a^2, $(1 - p_1)$ and $2p_1$ from (3) and substituting from (5) and (6) in expression (2), we ultimately find that the velocity-potential ϕ is given by

$$\phi = \dot{A}a \left\{(1 - p_1) F\left(-\frac{1}{2}, \frac{1}{2}, 1, r^2/a^2\right) + \frac{2}{3}p_1 F\left(-\frac{3}{2}, \frac{1}{2}, 1, r^2/a^2\right)\right\} \quad \dots\dots(7)$$

$$= \dot{A}a \left\{(1 - p_1) F_1 + \frac{2}{3}p_1 F_2\right\} \qquad \dots\dots(8).$$

The kinetic energy of the fluid associated with *both* sides of the disc is†

$$T = \rho \iint \phi \, \frac{\partial w}{\partial t} \, dS,$$

and since $dS = 2\pi r \, dr$, where dr is the width of an elementary annulus of radius r, we have

$$T = \rho \int_0^a \phi \dot{A} \left(1 - p_1 \frac{r^2}{a^2}\right) 2\pi r \, dr \qquad \dots\dots(9).$$

Inserting the value of ϕ from expression (8) in (9) above we obtain

$$T = 2\pi \rho \dot{A}^2 a \int_0^a \{(1 - p_1) F_1 + \frac{2}{3}p_1 F_2\} \left(1 - p_1 \frac{r^2}{a^2}\right) r \, dr \qquad \dots\dots(10).$$

It is easy to show that

$$\int_0^a F\left(\alpha, \beta, 1, \frac{r^2}{a^2}\right) r \, dr = \frac{a^2}{2} F(\alpha, \beta, 2) \qquad \dots\dots(11),$$

and that

$$\int_0^a F\left(\alpha, \beta, 1, \frac{r^2}{a^2}\right) r^3 \, dr = \frac{a^4}{2} \{F(\alpha, \beta, 2) - \frac{1}{2} F(\alpha, \beta, 3)\} \qquad \dots\dots(12).$$

Applying the relationships of (11) and (12) to the integral in (10) the kinetic energy can be written

$$T = 2\pi \rho \dot{A}^2 a^3 \left\{\frac{1}{2}(1 - p_1)\left[(1 - p_1) F\left(-\frac{1}{2}, \frac{1}{2}, 2\right) + \frac{1}{2}p_1 F\left(-\frac{1}{2}, \frac{1}{2}, 3\right)\right]\right.$$

$$\left. + \frac{1}{3}p_1 \left[(1 - p_1) F\left(-\frac{3}{2}, \frac{1}{2}, 2\right) + \frac{1}{2}p_1 F\left(-\frac{3}{2}, \frac{1}{2}, 3\right)\right]\right\} \qquad \dots\dots(13).$$

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* G. N. Watson, *Theory of Bessel Functions*, formula (2), p. 401 (1922).
† H. Lamb, *Hydrodynamics*, formula (4), p. 44.

Using Gauss's theorem for the reduction of the hypergeometric functions we get

$$T = 2\pi\rho\dot{A}^2 a^3 \left\{ \frac{(1-p_1)}{2} \left[(1-p_1)\frac{\Gamma(2)\,\Gamma(2)}{\Gamma(\frac{5}{2})\,\Gamma(\frac{3}{2})} + \frac{p_1}{2}\frac{\Gamma(3)\,\Gamma(3)}{\Gamma(\frac{7}{2})\,\Gamma(\frac{5}{2})} \right] \right.$$

$$\left. + \frac{p_1}{3}\left[(1-p_1)\frac{\Gamma(2)\,\Gamma(3)}{\Gamma(\frac{5}{2})\,\Gamma(\frac{5}{2})} + \frac{p_1}{2}\frac{\Gamma(3)\,\Gamma(4)}{\Gamma(\frac{7}{2})\,\Gamma(\frac{7}{2})} \right] \right\} \quad \ldots\ldots(14)$$

$$= \tfrac{8}{3}\rho a^3 \dot{A}^2 \left(1 - \tfrac{14}{15}p_1 + \tfrac{5}{21}p_1{}^2\right) \qquad \ldots\ldots(15)^*.$$

But
$$T = \tfrac{1}{2}M_1(\partial w/\partial t)^2 = \tfrac{1}{2}M_1\dot{A}^2,$$

where M_1 is the accession to inertia due to the fluid on *both* sides of the disc, and $\dot{A}$ is the central velocity.

Thus we find that

$$M_1 = \tfrac{16}{3}\rho a^3 \left(1 - \tfrac{14}{15}p_1 + \tfrac{5}{21}p_1{}^2\right) \qquad \ldots\ldots(16).$$

Rayleigh[†] has shown that the quantity $\tfrac{16}{3}\rho a^3$ is the accession to inertia for *both* sides of a rigid disc vibrating in an aperture in an infinite rigid plane—the distribution of radiation being spherical. Moreover $\left(1 - \tfrac{14}{15}p_1 + \tfrac{5}{21}p_1{}^2\right)$ represents the ratio of M_1 for a flexible to that for a rigid disc. When $p_1 = 0$ the rigid-disc formula is reproduced as we should expect, for then the amplitude at any radius is given by $w = A$, a constant. If $p_1 = 1$, $w = A\left(1 - r^2/a^2\right)$ and we have the case of a membrane vibrating in its gravest mode. The accession to inertia is $1\cdot625\rho a^3$, this being $0\cdot305$ that of a rigid disc.

For our particular case where the disc has one nodal circle, the integral of the momentum[‡] over the surface is zero when the radius r of this circle is $a/\sqrt{2}$ or $p_1 = 2$. Inserting this value in formula (16) we find that

$$M_1 = \tfrac{16}{35}\rho a^3 \qquad \ldots\ldots(17),$$

this being approximately 8·6 per cent of that for a rigid disc. The very large reduction in M_1 is due to the two equal areas on either side of the nodal circle vibrating in opposite phase.

By allotting any desired value to the parameter p_1, formula (16) can be used to ascertain the influence of the radius of the nodal circle on M_1 in relation to the central velocity $\dot{A}$. If $p_1 > 2$ the amplitude at the edge exceeds that at the centre. For this condition M_1 should be evaluated in terms of the edge velocity $\dot{A}(1 - p_1)$. Then $T = \tfrac{1}{2}M_1\dot{A}^2(1 - p_1)^2$, so that (16) must be divided by $(1 - p_1)^2$. For example, if the thickness of the disc varies as $(1 - r^2/a^2)$, p_1 for zero momentum is 3. The value of M_1 is identical with (17) but T is four times that for a uniform disc where $p_1 = 2$.

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* T is a minimum when $p_1 = 1\cdot96$.
† *Sound*, **2**, 162 (1894).
‡ This is the condition for a vibrational mode when the *edge is free* and the centre moving.

§3. FREE-EDGE DISC WITH ONE NODAL DIAMETER

Let the dynamic deformation curve be of the form

$$w = A \frac{r}{a} \cos \theta = \frac{A}{a} x \qquad \ldots\ldots(18),$$

where $x = r \cos \theta$ as in figure 2.

For finding the velocity-potential at points on the disc, equation (1) is barred by the asymmetry of (18), so we proceed as follows: When the dynamic deformation curve is of the type $w = A (1 - r^2/a^2)^{n+1}$, the velocity-potential from (1) is

$$\phi = \dot{A} \int_0^\infty J_0(kr) \, dk \int_0^a J_0(kr) \left(1 - \frac{r^2}{a^2}\right)^{n+1} r \, dr \qquad \ldots\ldots(19).$$

Infinite rigid plane

$w = A (r/a) \cos \theta$
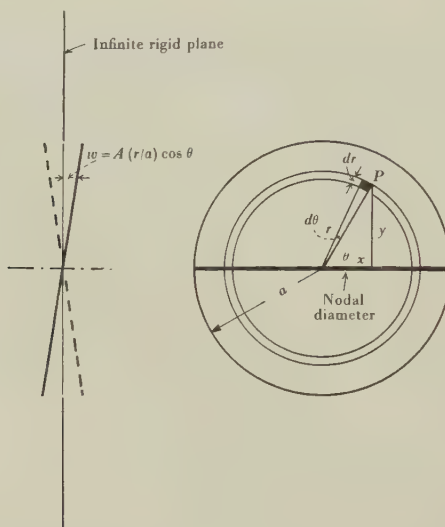
Nodal diameter

Fig. 2. Diagram illustrating flexible disc vibrating with one nodal diameter in an infinite rigid plane. The dynamic-deformation curve is $w = A (r/a) \cos \theta$, and the elementary area at P is $r \, dr \, d\theta$.

Substituting $r = a \sin \chi$ in the first integral of (19) and altering the limits accordingly, we have

$$a^2 \int_0^{\frac{1}{2}\pi} J_0(ka \sin \chi) \cos^{2n+3} \chi \sin \chi \, d\chi \qquad \ldots\ldots(20).$$

The form in expression (20) tallies with that of Sonine's first finite integral and its value* is

$$2^{n+1} \Gamma(n+2) a^2 \frac{J_{n+2}(ka)}{(ka)^{n+2}} \qquad \ldots\ldots(21).$$

Thus substituting (21) in (19) we obtain

$$\phi = 2^{n+1} \Gamma(n+2) \dot{A} a^2 \int_0^\infty \frac{J_0(kr) J_{n+2}(ka) \, dk}{(ka)^{n+2}},$$

* G. N. Watson, *Theory of Bessel Functions*, formula (1), p. 373 (1922).

and by formula (4) above this reduces to

$$\phi = \frac{\pi^{\frac{1}{2}}}{2}\frac{\Gamma(n+2)}{\Gamma(n+\frac{5}{2})}\, Aa\, F\left[-(n+\tfrac{3}{2}),\,\tfrac{1}{2},\,\mathrm{I},\,\frac{r^2}{a^2}\right] \qquad\ldots\ldots(22).$$

Since $r^2 = x^2 + y^2$ (figure 2), we find that*

$$\frac{\partial}{\partial x}\left\{A\left(\mathrm{I}-\frac{r^2}{a^2}\right)^{n+1}\right\} = -\,2\,(n+\mathrm{I})\frac{Ax}{a^2}\left(\mathrm{I}-\frac{r^2}{a^2}\right)^{n} \qquad\ldots\ldots(23).$$

Also, since $\quad \dfrac{\partial}{\partial x}\,F\,(\alpha,\,\beta,\,\gamma,\,z) = \dfrac{\alpha\beta}{\gamma}\,F\,[\alpha+\mathrm{I},\,\beta+\mathrm{I},\,\gamma+\mathrm{I},\,z]\,\dfrac{dz}{dx},$

we obtain $\qquad \dfrac{\partial\phi}{\partial x} = -\dfrac{\pi^{\frac{1}{2}}}{2}\dfrac{\Gamma(n+2)}{\Gamma(n+\frac{3}{2})}\dfrac{Ax}{a}\,F\left[-(n+\tfrac{1}{2}),\,\tfrac{3}{2},\,2,\,\dfrac{r^2}{a^2}\right] \qquad\ldots\ldots(24).$

Putting $n = 0$ in (23) we have $-\,2Ax/a^2$, whilst $n = 0$ in (24) yields

$$-\,\dot{A}\,(x/a)\,F\left(-\tfrac{1}{2},\,\tfrac{3}{2},\,2,\,r^2/a^2\right) \qquad\ldots\ldots(25).$$

Thus, if the deformation curve be of the form $A\,(x/a)$, the velocity-potential is found when (25) is multiplied by $-\,a/2$. Moreover, in this case

$$\phi = \frac{\dot{A}}{2}\,xF\left(-\tfrac{1}{2},\,\tfrac{3}{2},\,2,\,\frac{r^2}{a^2}\right) \qquad\ldots\ldots(26).$$

The kinetic energy associated with *both* sides of the disc is

$$T = \rho\int_0^a\int_0^{2\pi}\phi\left(\frac{\partial w}{\partial t}\right)r\,dr\,d\theta \qquad\ldots\ldots(27),$$

where $r\,dr\,d\theta$ is an elementary area on the disc (figure 2).

Substituting for the various quantities in the integrand of (27) and writing $x = r\cos\theta$ we have, for the kinetic energy,

$$T = \rho\,\frac{\dot{A}^2}{2a}\int_0^a F\left(-\tfrac{1}{2},\,\tfrac{3}{2},\,2,\,\frac{r^2}{a^2}\right)r^3\,dr\int_0^{2\pi}\cos^2\theta\,d\theta \qquad\ldots\ldots(28).$$

The first integral is π, whilst by expanding and integrating term by term the second integral is found to be $\dfrac{a^4}{4}\,F\left(-\tfrac{1}{2},\,\tfrac{3}{2},\,3\right)$. Substituting these values in (28) we have

$$T = \tfrac{1}{8}\pi\rho a^3\dot{A}^2\,F\left(-\tfrac{1}{2},\,\tfrac{3}{2},\,3\right) \qquad\ldots\ldots(29).$$

But $\qquad F\left(-\tfrac{1}{2},\,\tfrac{3}{2},\,3\right) = \dfrac{\Gamma(3)\,\Gamma(2)}{\Gamma(\frac{7}{2})\,\Gamma(\frac{3}{2})} = \dfrac{32}{15\pi}$, and therefore $T = \tfrac{1}{15}\dot{A}^2\rho a^3.$

Thus the accession to inertia for both sides of the disc is

$$M_1 = \tfrac{8}{15}\rho a^3 \qquad\ldots\ldots(30),$$

which is precisely $\tfrac{1}{10}$ that for a rigid disc.

* See N. M. Ferrers, *Quarterly Journal of Mathematics*, **14**, 7 (1877). This procedure involving differentiation is valid only when the amplitude at the edge of the disc is zero, before the derivative is obtained. Thus the original formula being $w = A\,(\mathrm{I}-r^2/a^2)^{n+1}$, the necessary condition is fulfilled, since $w = 0$ when $r = a$. It is not possible, however, to obtain the value of ϕ corresponding to $r\cos\theta$ by differentiating $w = A\,(r^2/a^2)$ with respect to x, since $w \neq 0$ when $r = a$. Obviously the differentiation can be extended to higher derivatives provided the above condition is satisfied.

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From formulae (17) and (30) we see that the ratio of the values of M_1 for a nodal circle and a nodal diameter is $\frac{80}{93}$ or 0.86. Moreover, there is not a wide divergence in M_1 for the two modes of vibration. In the above analysis the value $p_1 = 2$ was used, this corresponding to a nodal circle whose radius $r = a/\sqrt{2}$. Actually when there are no diametral nodes and the disc vibrates freely, $r = 0.68a$ and $p_1 = 2.16$. Using this value of p_1 in expression (16) and dividing by $(p_1 - 1)^2$, since $p_1 > 2$, we find $M_1 = 0.371\rho a^3$ and the ratio (nodal circle)/(nodal diameter) $= 0$.

§4. FREE-EDGE DISC WITH STATIONARY CENTRE

Let the dynamic deformation curve be of the form $w = A\,(r/a)^2$, figure 1 (b). Then the velocity-potential at points on the disc is, by (1),

$$\phi = \frac{\dot{A}}{a^2} \int_0^\infty J_0\,(kr)\,dk \int_0^a J_0\,(kr)\,r^3\,dr \qquad \ldots\ldots(31).$$

After a partial integration the first integral becomes

$$\int_0^a J_0\,(kr)\,r^3\,dr = a^4 \left\{ \frac{J_1\,(ka)}{ka} - \frac{2\,J_2\,(ka)}{(ka)^2} \right\} \qquad \ldots\ldots(32).$$

From the results in (5), (6) and (32), (31) gives

$$\phi = \dot{A}a\,\{F\,(-\tfrac{1}{2}, \tfrac{1}{2}, 1, r^2/a^2) - \tfrac{2}{3}\,F\,(-\tfrac{3}{2}, \tfrac{1}{2}, 1, r^2/a^2)\} \qquad \ldots\ldots(33)$$
$$= \dot{A}a\,\{F_1 - \tfrac{2}{3}F_2\} \qquad \ldots\ldots(34).$$

Thus the kinetic energy associated with *both* sides of the disc is given by

$$T = 2\pi\rho\,\frac{\dot{A}^2}{a^2} \int_0^a \phi r^3\,dr,$$

which on substitution for ϕ from (34) yields

$$T = 2\pi\rho\,\frac{\dot{A}^2}{a} \int_0^a (F_1 - \tfrac{2}{3}F_2)\,r^3\,dr \qquad \ldots\ldots(35).$$

The evaluation of (35) is accomplished by using the result given in (12), so that we obtain

$$T = 2\pi\rho\,\frac{\dot{A}^2}{a}\,\frac{a^4}{2} \left[F\,(-\tfrac{1}{2}, \tfrac{1}{2}, 2) - \tfrac{1}{2}F\,(-\tfrac{1}{2}, \tfrac{1}{2}, 3) - \tfrac{2}{3}F\,(-\tfrac{3}{2}, \tfrac{1}{2}, 2) + \tfrac{1}{3}F\,(-\tfrac{3}{2}, \tfrac{1}{2}, 3) \right],$$

which on reduction by Gauss's theorem gives the kinetic energy of the fluid as

$$T = \tfrac{40}{63}\,\dot{A}^2 \rho a^3 \qquad \ldots\ldots(36).$$

Thus the accession to inertia

$$M_1 = \tfrac{80}{63}\,\rho a^3 \qquad \ldots\ldots(37)^*,$$

which is $\frac{5}{21}$ or nearly $\frac{1}{4}$ that of a rigid disc. This exceeds the values obtained for a nodal circle or diameter, since in the present case the whole of the disc, at any particular epoch, is moving in the same direction. It is less than that for a rigid disc, since the amplitude of vibration gradually decreases from the periphery inwards.

* Dividing (16) by $(1 - p_1)^2$ and making $p_1 = \infty$ yields (37).

§ 5. INFLUENCE OF M_1 ON FREQUENCY OF VIBRATION

Since the curve of deformation *in vacuo* is maintained during vibration in the fluid, it follows that the potential energy is identical in both cases. Moreover, the kinetic energy of the system is alone affected and with it the frequency of vibration. The latter being inversely proportional to the square root of the total equivalent mass it follows that

$$\frac{f_1}{f_0} = \sqrt{\left(\frac{M_q}{M_q + M_1}\right)} = \sqrt{\left(\frac{1}{1 + M_1/M_q}\right)} \qquad \ldots\ldots(38),$$

where M_q is the equivalent mass of the disc during vibration *in vacuo*, M_q

  f_0 is the frequency *in vacuo*, f_0

and f_1 is the frequency in fluid. f_1

To estimate the magnitude of f_1/f_0 it is essential to determine the value of M_q.

§ 6. EVALUATION OF THE EQUIVALENT MASS M_q

(a) *Free-edge disc with nodal circle.* The kinetic energy of the disc *in vacuo* is $\Sigma \tfrac{1}{2}mv^2$ over its surface, and this is

$$\tfrac{1}{2}\rho_1\tau \int_0^a \dot{w}^2 \, 2\pi r \, dr = \frac{\pi\rho_1\tau a^2 \dot{A}^2}{2}\left(1 - p_1 + \frac{p_1^2}{3}\right) = \frac{\dot{A}^2}{2}M\left(1 - p_1 + \frac{p_1^2}{3}\right),$$

or $T_v = \tfrac{1}{2}\dot{A}^2 M_q = \tfrac{1}{2}\dot{A}^2 M k_q$ $\ldots\ldots(39),$

where T_v is the kinetic energy of the disc *in vacuo*, T_v

  M is the natural mass, M

  k_q, the mass coefficient, is $(1 - p_1 + p_1^2/3)^*$, k_q

  p_1 is the density of disc, and p_1

  τ is the thickness of disc. τ

Putting $p_1 = 2$ in the above, we find that for one nodal circle

$$M_q = \tfrac{1}{3}M \qquad \ldots\ldots(40),$$

and $k_q = \tfrac{1}{3}$.

(b) *Free-edge disc with one nodal diameter.* It will be of greater interest if we consider the general case of n diameters. Let the dynamic deformation curve be n
$w = A(r/a)^n \cos n\theta$. Then the kinetic energy T_v of the disc *in vacuo* is (using figure 2 and remembering that there are n diameters) given by

$$T_v = \frac{\dot{A}^2 \rho_1 \tau}{2a^{2n}} \int_0^a r^{2n+1} \, dr \int_0^{2\pi} \cos^2 n\theta \, d\theta$$

$$= \frac{\dot{A}^2}{2} \frac{\pi\rho_1\tau a^2}{2(n+1)},$$

so that the equivalent mass

$$M_q = M/2(n+1) \qquad \ldots\ldots(41),$$

and $k_q = 1/2(n+1)]$ $\ldots\ldots(42).$

For one nodal diameter $n = 1$, so that $M_q = \tfrac{1}{4}M$ and $k_q = \tfrac{1}{4}$. These formulae are

* When $p_1 > 2$, $k_q = (1 - p_1 + p_1^2/3)/(1 - p_1)^2$ as is shown in § 2. By making p_1 infinite the value of k_q in (c) is obtained.

also valid in the case of a conical shell where the deformation curve is as defined above.

(c) *Free-edge disc with stationary centre.* In this case $w = A\,(r/a)^2$ and, therefore,

$$T_v = \frac{\dot{A}^2 \pi \rho_1 \tau}{a^4} \int_o^a r^5 \, dr$$

$$= \tfrac{1}{2} \dot{A}^2 \pi \rho_1 \tau a^2 / 3,$$

whence
$$M_q = \tfrac{1}{3} M \qquad\qquad\qquad \ldots\ldots(43),$$

and
$$k_q = \tfrac{1}{3}.$$

Consequently the values of M_q and k_q for a free-edge disc with one nodal circle and with stationary centre are identical.

It is important to observe that the *equivalent* mass is quite distinct from the *effective** mass. The former refers to the whole surface and is associated with energy relationships; it is always positive and never zero. The latter is referred to the driving point on the disc and is associated with the mechanical impedance. It can be positive, negative or zero.

§7. COMPUTATION OF THE RATIOS M_1/M_q AND f_1/f_0

$$\frac{M_1}{M_q} = \frac{k_1}{k_q} \frac{\rho}{\rho_1} \frac{a^3}{\pi a^2 \tau}$$

$$= \frac{1}{\pi} \frac{k_1}{k_q} \frac{\rho}{\rho_1} \frac{a}{\tau} \qquad\qquad \ldots\ldots(44),$$

k_1

where the accession coefficient k_1 is (M_1 for flexible disc) $\div$ (M_1 for rigid disc).

To illustrate the foregoing analysis concretely we shall calculate the ratios M_1/M_q and f_1/f_0 for the cases (a), (b), (c) in air and in water. Taking an aluminium disc for which $a = 10$ cm., $\tau = 0\cdot055$ cm., density of aluminium $\rho_1 = 2\cdot7$ gm./cm^3, density of air $= 1\cdot3 \times 10^{-3}$ gm./cm^3 and of water $= 1\cdot0$ gm./cm^3, we obtain the data set forth in tables 1 and 2 below.

Table 1. Vibration in air

Type of vibration	M_1/M_q	Reduction in frequency
		%
One nodal circle (a)	0·038	1·9
One nodal diameter (b)	0·059	3·0
Stationary centre (c)	0·106	5·3

Table 2. Vibration in water

Type of vibration	M_1/M_q	Frequency ratio
One nodal circle (a)	29·2	0·182
One nodal diameter (b)	45·4	0·147
Stationary centre (c)	81·6	0·111

* *Proc. Phys. Soc.* **44**, 88 (1932).

The data of table 1 indicate that the alteration in frequency in air is relatively unimportant from a practical standpoint. On the other hand, the results in table 2 are somewhat startling to anyone unversed in the subject of subaqueous acoustics. In case (a) the accession to inertia is 29·2 times the equivalent mass of the disc and nearly ten times its natural mass. If the frequency f_0 *in vacuo* were 120 $\sim$, the frequency f_1 in water would fall to 21·8 $\sim$! If the disc were made of steel, its mass for the same value of f_0 would be three times that of the aluminium disc, so that in this case $f_1 = 21\cdot8 \sqrt{3} = 36\cdot7 \sim$. From expression (44) it is evident that if the radius is constant, increases in either the density of the disc or its thickness, or both, entail a corresponding rise in the frequency ratio f_1/f_0. In other words, the higher the frequency *in vacuo*, the smaller the reduction, due to accession to inertia, in a fluid.

THE DETERMINATION OF REFRACTIVITY TEMPERATURE COEFFICIENTS FOR LIQUIDS

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ABSTRACT. In this paper attention is directed chiefly to the problem of measuring with precision the changes in the refractivity of a liquid for small alterations in temperature. An elaborated Jamin interferometer is described, as are also the several pieces of auxiliary apparatus necessary for setting up and maintaining differences in the temperature of the two interferometer tubes. Further, there is given a plan for measuring, by means of platinum resistance thermometers, differences in the temperature of the two tubes.

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§ 1. INTRODUCTION

DURING a recent research, certain variations in the refractivity of benzene were determined with the aid of a Jamin interferometer. As the work progressed the instrument was, as described elsewhere*, in some degree elaborated and increased accuracy imparted to the required measurements. In this communication a description is given of additional devices (applicable to interferometers of the Jamin type), which facilitate the determination of refractivities with a precision not otherwise readily attained. To begin with we briefly review certain difficulties inherent in such interferometric methods.

First, we observe that in determining a refractivity there is, in general, some uncertainty as to the true temperature of the liquid. This is of little moment when the measurements are made in a laboratory having a convenient and practically constant temperature, and the influence of draughts is excluded by surrounding the containing tubes with an aluminium case; for under these circumstances both tubes are equally affected. But for the determination of a refractivity temperature coefficient, some definite difference in the temperatures of the contents of the tubes must be established and maintained. Failing this, some plan for securing a constant difference in the two temperatures is allowable. Such a plan does not necessarily involve a strict constancy in the two temperatures; but it does require that, for a given difference, any small variation in the temperature of the one tube shall be accompanied by a precisely similar, equal and simultaneous variation in that of the other.

Assuming that this difficulty is overcome, we are at once confronted with the further problem of determining with accuracy (*a*) the temperature of the contents of each tube; or (*b*) the difference in the temperature of the two liquids.

* *Phil. Mag.* **13**, 249–64 (1932).

For the solution of this problem two methods are at once presented. According to the one, thermometers are introduced through the tubulures of the interferometer chambers; whilst for the second, other thermometers, having exceptionally long and narrow bulbs, are placed horizontally upon the chambers. But from experiments it was found that both plans are inconvenient and lacking in precision. An attempt to win success by surrounding the interferometer chambers with water jackets failed, owing to the impossibility of circulating the water with the necessary rapidity. Even with the quickest flow obtainable, a distinct temperature-gradient was observable between the extremities of the jacket. Theoretically, success by this simple method can be achieved only by establishing an infinitely rapid circulation of the water. The difficulty was evaded in the following way.

§ 2. APPARATUS

First, a new interferometer, having a gap for tubes some 60 cm. long, was designed and built. The glass prisms of this instrument were such that the interfering beams of light were widely separated. When the instrument was received the usual glass tubes were displaced by others of copper having walls 1 mm. thick.
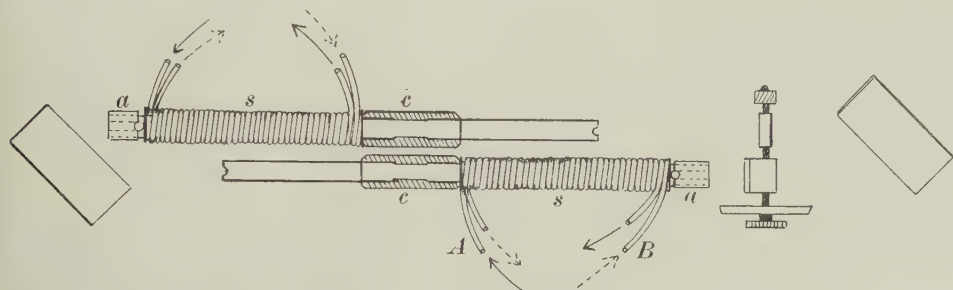

Fig. 1.

Secondly, each copper tube was cut into two unequal lengths which were then trued in a lathe. Subsequently these were reunited with a tightly fitting boxwood cylinder *c*, figure 1, so that the enclosed ends were 15 mm. apart as indicated in the figure.

Thirdly, the longer limb of each compound tube was, as shown in figure 1, very closely and simultaneously wound with twin lengths of "compo" tubing, the bore of which was 2 mm. Then by means of tape the tubular spirals *s*, *s* were firmly bound to the encircled copper tube. Finally they were covered with bright tinfoil and radiation effects were thus reduced to a minimum.

Lastly, the metal pillars provided for supporting the interferometer tubes were somewhat shortened and then fitted with thin adjustable plates of non-conducting material. In these plates were cut the necessary **V** grooves for the reception of the compound tubes or jackets (*vide supra*).

Given the above-described apparatus and confining ourselves to temperatures not greatly removed from that of the laboratory, we can readily set up and maintain any one of a series of small and constant temperature-differences as between and

within the two compound interferometer chambers. That this is so, will at once be seen from the following considerations.

Referring again to figure 1, let a stream of water constant in temperature and velocity pass through the spiral A in the direction shown. Under these conditions the extremities of the enclosed copper tube attain certain invariable temperatures θ_1, θ_2; and for small differences the gradient may be represented by ab, figure 2, α. When the other spiral also is used alone, and the experiment repeated by passing the water in at B, the temperature-gradient will be cd. On adding the opposed but precisely equal gradients, we obtain the result represented by the dotted graph mn; whence it follows that by operating the two spirals simultaneously and in opposite directions, we attain the desired end and secure uniformity in temperature throughout the entire length of the interferometer chamber. This is for small differences dt. But when the difference $\theta_1 - \theta_2$ is large, the result is that represented in figure 2, β. Here it is seen that for the central section of the tube the temperature is lower than that of the ends. When, however, $\theta_1 - \theta_2$ does not exceed $5°$ C., any inequality of temperature becomes, owing to the high conductivity of copper, vanishingly small and altogether negligible. In the absence of the copper
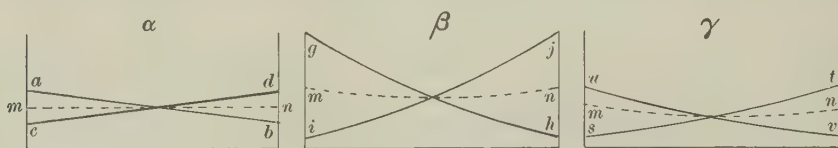

Fig. 2.

tube, the distribution of temperature along the operating spirals is, for the small differences under consideration, represented as in figure 2, γ. The plan adopted for realizing this theoretical uniformity and constancy in the temperatures of the interferometer chambers was as follows.

First, two thermostats were set up and conveniently placed, the one for serving the twin spirals of the first chamber and the other for serving those of the second. The temperature of each thermostat was indicated by a Baudin thermometer graduated in $\frac{1}{50}°$ C., and easily readable to $\frac{1}{200}°$ C. With the aid of regulators charged with xylene, the usual electrical devices, and centrifugal stirrers, the two temperatures were maintainable within limits approximating to $\pm \cdot 005°$ C.

Secondly, each thermostat T was fitted with two siphons p, p of "compo" tubing, and an air injector J as shown diagrammatically in figure 3. The inlets of the twin spirals were connected, the one with the first siphon and the other with the second, whilst the outlets were similarly attached to the tubes t, t of the injector. The vertical limb L was led away and joined to the air compressor shown in the explanatory figure 4.

Thirdly, in each case the temperature of the thermostat was adjusted and the apparatus made ready for action by first closing the stopcock c and the side-tube s, and then applying suction to the tube e. The siphons were thus activated and the spirals filled with water.

Lastly, a stream of air governed by the tap *c* was admitted from the compressor; and this on its way through *J* caused an intermittent flow of water through the spirals.
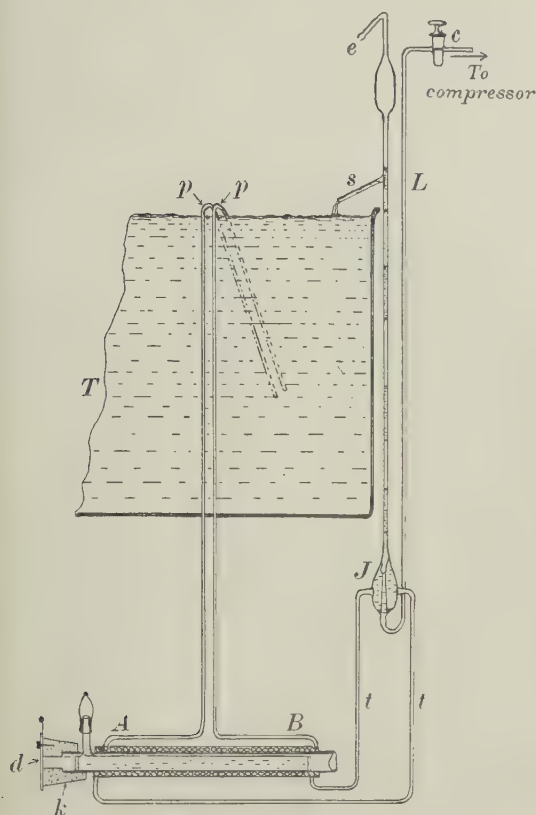

Fig. 3.
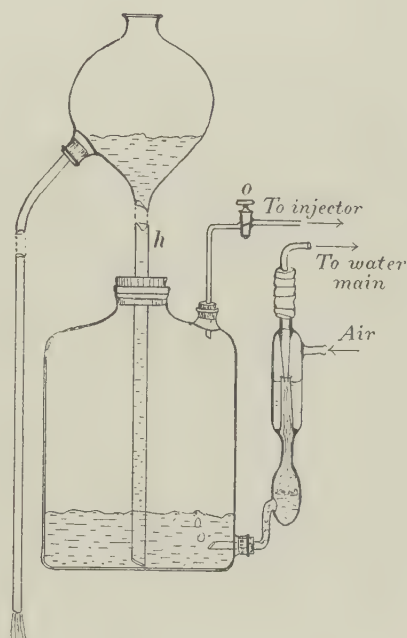
Fig. 4.

It may be noted that the temperature to which an interferometer chamber is raised depends largely upon the rate of flow; the maximum increase being obtained when the rate also is a maximum. Thus for each given rate there is a corresponding constant temperature. In illustration of this, the following data are here tabulated:

Air stream	Temperature of thermostat (° C.)	Temperature of chamber (° C.)	Temperature of laboratory (° C.)
Passing slowly	19·7	15·7	13
,, moderately slowly	19·7	16·5	13
,, rapidly	19·7	17·7	13

The apparatus having been completed, it was deemed advisable to obtain experimental proof of the correctness of the conclusions, reached theoretically, as to the uniformity of the temperature within the interferometer chambers.

Accordingly use was made of a thermoelectric couple consisting of fine wires of nickel and iron, and a low resistance reflecting galvanometer. The couple having been standardized, the galvanometer and its scale were so adjusted that the minimum reading obtainable corresponded to $\frac{1}{100}°$ C. The following experiments were then carried out.

The temperature of one of the thermostats was raised to 19° C., the couple was placed centrally within and against the wall of the chamber, and the water was passed continuously and rapidly through the spirals. When the deflection D D was constant, the couple was slowly moved along within and against the chamber and the temperature thus tested from the one extremity of the spirals to the other. At first very distinct variations in D corresponding in some instances to nearly 0·1° C. were observable. But finally these died away and a common value for D obtained for every part of the chamber. Hence theory and experiment were here in complete accord. We now describe the method used for determining with precision (*a*) the difference in the temperature of the two interferometer tubes, and (*b*) the temperature of each. Of these the former is for our present main purpose the more important.

The inconvenience attending the use of mercury thermometers, to which we have already alluded, led to the employment of platinum resistances. Apart from their sensitivity and dead-beat character, such instruments have great adaptability; and provided the worker possesses the requisite skill and technique, no insuperable difficulty is met with in constructing platinum thermometers capable of satisfying any legitimate demand.

The glass containing chambers of the interferometer were 20 cm. long and of the form shown in figure 5. And in order that the chambers might be used for almost any liquid, the windows were of the adhesed or fused-on type. Upon each chamber was laid a bare platinum wire arranged loop-wise, and nearly equal in extent to
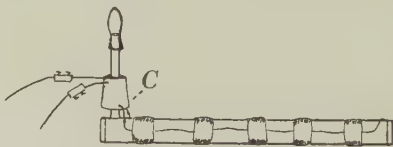

Fig. 5.

the length of the chamber. To the ends of the wire, which was 0·03 mm. in diameter, gold leads were fused; and these received ample support from a large cork C through which they were threaded. The arranged platinum loop was, as shown in the figure, secured to the chamber by means of narrow white ribbon, the ends of which were fastened by "Durofix" cement.

Prior to the fixing *in situ* of the thermometers, data for the calculation of their constants were obtained by successively measuring the resistance R of one of the wires when the wire was placed (*a*) in melting ice, (*b*) in steam, and (*c*) in sulphur vapour. It was thus found that in the expression $R_t = R_0 (1 + \alpha t - \beta t^2)$, $\alpha = 3926 \times 10^{-6}$ and $\beta = 5956 \times 10^{-10}$. (Incidentally, these values indicate that

the wire was, as stated by the maker, "exceptionally pure.") We observe that for small changes in temperature the influence of the constant β is wholly negligible.

At $0°$ C., the resistance R_0 of thermometer No. 1 was $6.5824\,\Omega$, whilst that of No. 2 was $6.6349\,\Omega$. At $1°$ C. the resistance of No. 1 was increased by $0.0263\,\Omega$. By trial it was found that variations in R as small as $4 \times 10^{5}\,\Omega$ were determinable. Hence a difference in temperature equal to $0.002°$ C. was detectable.

§ 3. PROCEDURE

Having now set forth certain essential details, we next briefly describe the procedure observed in measuring a refractivity temperature coefficient. It is as follows.

First, the glass interferometer chambers are each charged with portions of the given liquid, then closed with their capsules and introduced into their respective and closely-fitting water-jackets shown in figure 1. The ends of the jackets having been covered in the way indicated in figure 3*, the thermostats, both at the temperature of the laboratory, are brought into use, and the water is circulated in the two sets of spirals. Later, observations are taken at short intervals. In general, owing to the changing temperature, the interference fringes are at first hazy, somewhat indefinite or even markedly wavy. Finally, however, they become sharp and stationary, thus showing in the most decisive manner possible that equilibrium of temperature has been attained. The platinum thermometers are now electrically balanced against each other and the bridge reading r_1 noted.

Next, the temperature of one of the two thermostats is suitably increased, say by $3°$ or $4°$ C., and a second set of observations carried out and the new bridge reading r_2 obtained. Knowing now the value of $(r_1 - r_2)$, we can, with the aid of the allied constants, at once calculate the difference in the temperature of the two chambers and therefore of their contents. Given this temperature-difference and the corresponding shift in the fringes, the sought-for refractivity temperature coefficient may then be found†.

When the value of a coefficient k is required for temperatures higher than those indicated above, both thermostats are first set for the chosen minor temperature and brought under automatic control. The necessary data having been obtained, the temperature of one of the two thermostats is then increased by some $3°$ or $5°$ C., and a second set of observations is carried out as already described: the value of k may then be calculated.

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* The caps of the copper tubes are of cork covered with tinfoil, and the pivoted shutter of polished aluminium. The success of the device depends, as will be seen, upon the high non-conductivity of cork and the high reflectivity of the two metals for heat.

† The success of this plan, in which the platinum thermometers are used differentially, largely depends upon a fairly close equalization of the two resistances r_1, r_2. Failing this, the ratio r_1/r_2 will vary as the common temperature varies. In the present instance $(r_1 - r_2) = 0.05\,\Omega$; and the ratio $r_1/r_2 = 1.007976$. For a range of $10°$ C. this ratio is, as calculation shows, quite unaffected.

§4. CONCLUSION

It will be seen that the differential method now advanced and detailed is theoretically, entirely correct. For although the temperature of the cooler thermostat may and probably does to some extent vary whilst its companion is being warmed, the only effect is one that either increases or diminishes the final difference in the temperature of the two interferometer chambers. That this is so follows from the fact that the ratio of the two thermometer resistances is, as we have shown, invariable.

When circumstances are such that a knowledge of the actual temperature of each chamber is indispensable, the resistance thermometers are, for convenience, joined to the usual measuring apparatus by an appropriate mercury switch, and the necessary data obtained by well-known methods.

In a future communication our purpose will be to give an account of an investigation for which the interferometer dealt with here has been designed and built.

In conclusion, it may be remarked that the apparatus used for maintaining the two interferometer chambers at various constant temperatures is equally applicable to the tubes of polarimeters and the like.

A METHOD FOR DEDUCING ACCURATE VALUES OF THE LATTICE SPACING FROM X-RAY POWDER PHOTOGRAPHS TAKEN BY THE DEBYE-SCHERRER METHOD

By A. J. BRADLEY, Ph.D., and A. H. JAY, M.Sc.

ABSTRACT. A method has been tried for obtaining accurate values of lattice spacings from X-ray powder photographs taken in the usual circular type of camera. There are two essential features of the method, (*a*) the calibration process, (*b*) the extrapolation process.

The calibration process: The exposed portion of the film is limited by sharp knife edges, and the length S_k is measured for each film at the same time as the distance S between corresponding pairs of lines. S_k corresponds to an angle θ_k in the same way that S corresponds to a glancing angle θ, so that $\theta_k = S_k/4R$, where R is the radius of curvature of the film. R is an uncertain quantity depending on the amount of film-shrinkage, and so in calculating θ from S we replace R by θ_k, which is a constant for a particular camera. θ_k having been determined in a preliminary experiment, it is possible to calculate θ for a given reflection, from the formula

$$\theta = \frac{S}{S_k}\theta_k.$$

Errors due to film-shrinkage are hereby eliminated.

The extrapolation process: Other errors due to absorption by the specimen and eccentricity of the specimen may be eliminated by plotting the values of the lattice spacing calculated from a given pair of lines, against the corresponding values of $\cos^2\theta$. For small values of $\cos^2\theta$, the curve is almost linear, and is easily extrapolated to $\cos^2\theta = 0$, where the correct value of the lattice spacing is to be found. Only a few accurate measurements at angles where $\cos^2\theta$ is small are necessary in order to carry out this process.

As examples of the method, results are given for the lattice spacing of iron taken in different cameras with specimens of different diameters. They are consistent to 1 part in 15,000, the mean value being 2·8605. A specimen of electrolytic nickel (3·5162 Å.) was found to give a different value from a specimen of Mond nickel which had been degassed (3·5170 Å.).

§ 1. INTRODUCTION

I T is often necessary to obtain accurate values of lattice spacings, while at the same time a complete powder photograph is required. If the Debye-Scherrer method is used with the usual type of circular camera, it is difficult to make sure of the accuracy of the results since there may be a large number of systematic errors. It is possible to correct for each known source of inaccuracy, but in practice this is troublesome and the results are uncertain. A more satisfactory method has been described by Kettmann*. A value of the lattice spacing is calculated for each line on the film. These values are plotted against the corresponding value of the

* G. Kettmann, *Z. f. Phys.* **53**, 198 (1929).

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θ   glancing angle θ, and a smooth curve is drawn through the points so obtained. On extrapolating this curve to $\theta = 90°$ the true value of the spacing is obtained.

The form of the Kettmann curve is somewhat uncertain as it depends on the nature of the errors, but under certain conditions it is possible to reduce the number of errors to such an extent that the process of extrapolation is no longer arbitrary. The present paper describes a method of calibration which in many instances removes the need for correction. Any systematic errors which may occur can be easily eliminated by plotting the values of the spacing against $\cos^2 \theta$, whereupon the process of extrapolation is greatly simplified, as will be seen from the following account.

§2. THE METHOD OF CALIBRATION

In the Debye-Scherrer type of camera the powder specimen to be examined is placed at the centre of the camera, and the diffracted beams spreading out from this specimen strike the cylindrical film wrapped round the circumference of the camera. The object of the experiment is to determine the angle (2θ) between the direction of the incident beam and that of the diffracted beam, and so to find the

a   lattice spacing (a) of the powdered crystal. For a cubic crystal the lattice spacing is finally calculated from the formula

$$a = \lambda \sqrt{(h^2 + k^2 + l^2)}/2 \sin \theta \qquad \text{......(1)},$$

λ   where λ is the wave-length of radiation used and θ is the glancing angle of incidence
h, k, l   for the plane (hkl).

From the above equation it may be seen that the accuracy with which it is possible to determine a depends on the knowledge of λ and evaluation of θ. The accuracy with which λ is known is in most cases extremely high owing to the work of Siegbahn[*] and others. For this reason the accuracy of a is determined almost

S   entirely by θ, which may be obtained from a measurement of the distance (S) between corresponding pairs of lines at opposite ends of the photographic film,
R   and by ascertaining the film radius R. Then

$$S = 4R\theta \qquad \text{......(2)}.$$

It is difficult to apply the above formula directly since the value of R which satisfies it varies with each photograph, depending on the thickness of the photographic film and the manner in which the film is handled.

This difficulty is especially enhanced by the uncertain amount of contraction which occurs during the handling of the film, from the time it is placed in the camera until the time it is measured, this contraction being equivalent to a reduction in R. The uncertainty is eliminated by using a method of calibrating the film which permits the replacement of a fixed value R by a value which allows for the contraction in each case. Two types of method are available. In *the mixture method*[†]

* Siegbahn, *Spektroskopie der Röntgenstrahlen* (1931).
† Havighurst, Mack and Blake, *J. Amer. Chem. Soc.* **46**, 2368 (1924). F. C. Blake, *Phys. Rev.* **2**, 26, 60–70 (1925).

the powder is mixed with a substance of known lattice constant. The relation between the spacings and measured values of S for the standardizing substance is plotted graphically, and the values for the substance under investigation may be read off. This has the advantage of eliminating all systematic errors simultaneously, but the film is less easy to interpret.

As an alternative *the circumference of the camera may be marked in some way to correspond to definite values of* θ. These marks then provide a means of correlating values of θ and S without requiring a direct determination of R.

We have employed a modification of this method which will now be described. The camera is provided with two stops A and B, figure 1, which provide permanent points of reference. These stops form the two ends of the exposed portion of the film. The length S_k of the exposed portion of the film AYB corresponds to an angle $4\theta_k$, defined by

$$S_k = 4R\theta_k \qquad \ldots \ldots (3),$$

S_k

θ_k

R being the "virtual radius" of the camera as defined by equation (2). It is less than the true radius owing to film shrinkage. Combining equations (2) and (3) we get

$$\frac{\theta}{S} = \frac{\theta_k}{S_k} \qquad \ldots \ldots (4).$$

Instead of the uncertain value R, we now have the fixed angle θ_k, which is a constant for a particular camera. The value of S_k is carefully measured on every film at the same time as S. It is then possible to obtain θ from S, provided θ_k is known, and the film shrinkage is automatically allowed for.

The calibration of the camera thus consists in determining the angle θ_k. This is a constant for a particular camera, and need only be determined once. It is most important to ascertain this angle with accuracy, since it enters into all further work. For this purpose two methods have been used.

In the first method the camera is measured up directly. We require the ratio of the arc AYB to the radius R. It is more accurate to measure the smaller arc AXB, figure 1. Then

$$\theta_k = (\tfrac{1}{2}\pi - AXB/4R).$$

Since θ_k exceeds $80°$, the percentage error in θ_k is far less than the percentage error in the measurement of AXB. Also, since AXB is far smaller than $2R$, the accuracy of AXB ultimately determines the accuracy of θ_k.

Alternatively, a photograph may be taken of a substance of known lattice spacing, and the positions of the lines on the film can then be measured, together with the length S_k. Each line corresponds to a given value of θ and we may thus calculate a series of values for θ_k by means of equation (4). This method would, under suitable conditions, be the most satisfactory way of obtaining the value of θ_k. In practice difficulties may arise, which sometimes makes it easier to use the first method. It has been found that the results obtained by the two methods agree in general to about 1 part in 4000 for a camera of diameter 9 cm.

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Table 1 gives the data from which a camera was calibrated by the second method. Pure chemically prepared NaCl was used as the standardizing substance. This has been shown by Goldschmidt* to have a lattice spacing 5·626 Å. which is smaller than the value 5·628 Å. accepted for rocksalt. The value 5·626 Å. gives a constant series of values for θ_k. We show in the next section that this is an important confirmation of the correctness of the value. The value of θ_k found by this means was 80·945, which agrees closely with the value 80·96 obtained by the first method.

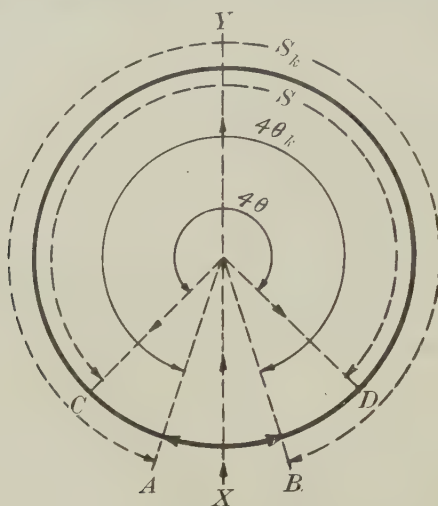Fig. 1. Illustrating method of calibration.

Table 1. Calibration by NaCl (spacing 5·626 Å.)

Σh^2	hkl	Radiation	S (observed)	θ (calculated)	θ_k or $\dfrac{S_k}{S}\theta$	Mean value of θ_k for each α doublet
End of film	—	—	25·651 (S_k)	—	—	—
52	640	α_1	25·394	80·135	80·946	80·946
51	$\begin{cases}711\\551\end{cases}$	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	24·718 / 24·509	77·999 / 77·342	$\left.\begin{array}{l}80·943\\80·946\end{array}\right\}$	80·945
48	444	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	22·699 / 22·555	71·612 / 71·185	$\left.\begin{array}{l}80·925\\80·956\end{array}\right\}$	80·941
44	622	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	20·695 / 20·596	65·304 / 64·994	$\left.\begin{array}{l}80·943\\80·946\end{array}\right\}$	80·945
40	620	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	19·029 / 18·936	60·027 / 59·779	$\left.\begin{array}{l}80·916\\80·977\end{array}\right\}$	80·947
					Mean:	80·945

The method of calibration just described eliminates errors due to film shrinkage and simultaneously fixes a point on the camera for which the glancing angle is known with extreme accuracy. This point is chosen at an angle in the region where

* V. M. Goldschmidt, *G.V.* 8 (1927).

the α doublet is well resolved, and near a point for which there are in many instances several lines suitable for measurement. Naturally where there is a choice of radiations available, a wave-length will be selected which gives a reflection close to this reference point. For example, with α iron using Co radiation we get K_a lines for (310) at 80° or 81°. Hence with a reference point at about 82° extremely accurate measurements of lattice spacing are obtainable merely by the use of this one doublet. In some cases it is not possible to obtain lines so near the end of the film. For example, copper with Copper K_a radiation does not give an α reflection beyond 73°. In this case the measurement may be influenced by various sources of error, which, if allowed to go unchecked, would vitiate the accuracy of the results. A brief account will be given of some of the causes of trouble, and it will be shown that a simple and straightforward method of treatment which practically eliminates every difficulty may be followed.

§ 3. SOURCES OF ERROR IN DETERMINING LATTICE SPACINGS

The possible sources of inaccuracy may be deduced from equation (1). We see that if da is the error in determining a,

$$\frac{da}{a} = \frac{d\lambda}{\lambda} - \cot\theta \, d\theta \qquad \ldots\ldots(5),$$

where $d\lambda$ is the error in λ, and $d\theta$ the error in θ. Assuming $d\lambda$ to be negligible, we have

$$da/a = -\cot\theta \, d\theta \qquad \ldots\ldots(6).$$

$d\theta$ can be found from equation (4), as follows,

$$\frac{d\theta}{\theta} = \frac{d\theta_k}{\theta_k} - \frac{dS_k}{S_k} + \frac{dS}{S} \qquad \ldots\ldots(7),$$

where $d\theta_k$, dS_k and $d\theta$ are the errors in θ_k, S_k and θ respectively.

The errors in θ_k arise in the process of calibration. It has been shown that in the first method of calibration by measuring the arc AXB, figure 1, the value of da is virtually proportional to dS_k alone, the exact value found for the radius being comparatively unimportant.

In the second method of calibration, where a standard substance is employed, there are possibilities of systematic errors. It is possible that a somewhat incorrect lattice spacing may have been assumed for the substance. This error may arise if NaCl is used, since the chemically prepared compound has not the same spacing as natural rocksalt. The effect of this error is shown in table 2, in which we have used an incorrect value (5·632) for the lattice spacing, obtaining a series of values for θ_k which vary considerably. In figure 2 we have plotted the value of θ_k against the appropriate value of θ for each line on the film. This has been done for various values of a. Only in one case do we get a constant series of values for θ_k, viz. when the correct value of the spacing has been employed. This provides a useful check on the method.

Table 2. θ_k calculated from an incorrect value of the lattice spacing (5·632)

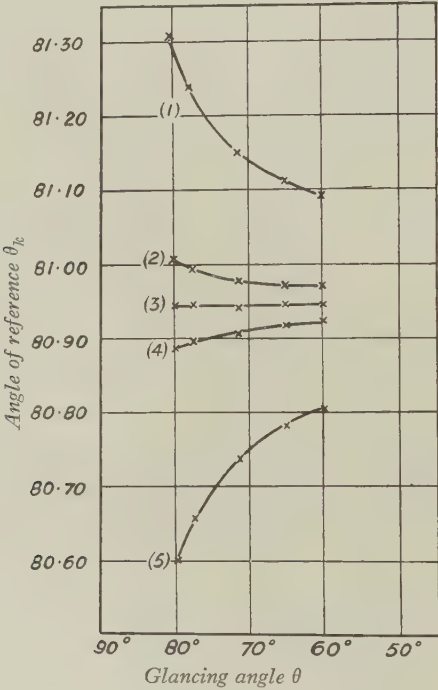| Σh^2	hkl	Radiation	S (observed)	θ (calculated)	θ_k or $\dfrac{S'}{S}\theta$	Value of θ_k for each α doublet
End of film	—	—	25·651	—	—	—
52	640	α_1	25·394	79·790	80·598	80·60
51	$\begin{cases}711\\551\end{cases}$	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	24·718 24·509	77·716 77·073	$\left.\begin{matrix}80·649\\80·664\end{matrix}\right\}$	80·66
48	444	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	22·699 22·555	71·429 71·006	$\left.\begin{matrix}80·718\\80·753\end{matrix}\right\}$	80·74
44	622	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	20·695 20·596	65·172 64·864	$\left.\begin{matrix}80·779\\80·784\end{matrix}\right\}$	80·78
40	620	$\begin{cases}\alpha_2\\\alpha_1\end{cases}$	19·029 18·936	59·921 59·674	$\left.\begin{matrix}80·773\\80·835\end{matrix}\right\}$	80·80

Fig. 2. Assumed lattice spacing: (1) 5·620; (2) 5·625; (3) 5·626; (4) 5·627; (5) 5·632.
The correct, constant, value of θ_k is given by (3).

It is of course essential to ensure that no errors be introduced due to bad centering of the specimen, absorption, etc. Each of these errors tends to make θ_k inconstant, but the nature of the variation of θ_k depends on the type of error. By using a very thin specimen of non-absorbent material such as NaCl the absorption difficulty may be eliminated, and it is also advisable to check the results by the first method.

With the best possible precautions, the limit of accuracy for θ_k is set by S_k, and we may replace $d\theta_k/\theta_k$ by dS_k (calibration)$/S_k$ (calibration), which is the error introduced in the actual measurement of S_k during calibration. Equation (7) may now be written:

$$\frac{d\theta}{\theta} = \left(\frac{dS_k \text{ cal.}}{S_k \text{ cal.}} - \frac{dS_k}{S_k}\right) + \frac{dS}{S} \qquad \ldots\ldots(8).$$

The expression in brackets represents the uncertainty in the position of the reference points.

The errors in S represented in equation (8) are of varying degrees of importance. Individual errors may arise from bad readings due to faint or fuzzy lines. These may be greatly minimized by measuring up a number of lines, but in the case of systematic errors another method of treatment must be adopted. Before describing this method we may first mention two points of minor importance, which may nevertheless in extreme cases affect the accuracy of the results.

The length of specimen in the beam influences the breadth of the line, causing a broadening of the lines at both small and large glancing angles. If measurements are made to the centre of blackening of the line, we get the two following expressions:

$$d\theta = -\frac{1+x}{96}\left(\frac{h}{R}\right)^2 \cot 2\theta \qquad \ldots\ldots(9),$$

$$\frac{da}{a} = \frac{1+x}{192}\left(\frac{h}{R}\right)^2 (\cot^2 \theta - 1) \qquad \ldots\ldots(10),$$

where h is the vertical divergence of the reflected beam, R is the radius of the camera, and x is a fraction between 0 and 1 depending on whether the beam fades at the edges or is uniform.

The expression $(\cot^2 \theta - 1)$ varies from infinity at $\theta = 0°$ to zero at $\theta = 45°$, when it changes sign and reaches the value -1 at $\theta = 90°$. It will be seen that if h/R is small the error is negligible except at very low angles. With fairly large values of h/R the calculated lattice spacing will be slightly depressed at high angles. It is important to keep down the value of h/R, in order to avoid this effect.

If the film shrinks unevenly or is not placed correctly in the camera, errors are introduced which may be considerable near the centre of the film, but will vanish at angles near the fixed reference point. Consequently most reliance should be placed on results obtained from measurements near this point. This is conveniently done by the method of extrapolation to be described below.

These errors are trivial compared with those due to the two main sources of inaccuracy which have now to be considered. These are (i) eccentricity of the specimen, (ii) absorption in the specimen.

(i) *Error due to eccentricity of specimen*

Let XY be the direction of the incident beam, which is assumed to be parallel, figure 3 *a*. Suppose A to be the true centre of the camera, but the specimen to have been placed at B. Let $AB = p$, and let the angle YAB be ϕ. All distances

on the circumference are reckoned positive away from Y. Then the displacement AB may be resolved into two components:

AB_1 or $p \cos \alpha$ in the direction of the incident beam, figure 3 b.

AB_2 or $p \sin \alpha$ perpendicular to the incident beam, figure 3 c.

It may be seen from figure 3 b that the displacement AB_1 introduces an error into the measured distance S between any given pair of lines. In this figure R_A and R_{A}' are the correct positions of the lines; they are displaced to R_B and R_{B}' by the displacement AB_1 of the specimen. Then if dS is the error in S,

dS

$$dS = 2R_A R_B = -2AB_1 \sin 2\theta$$
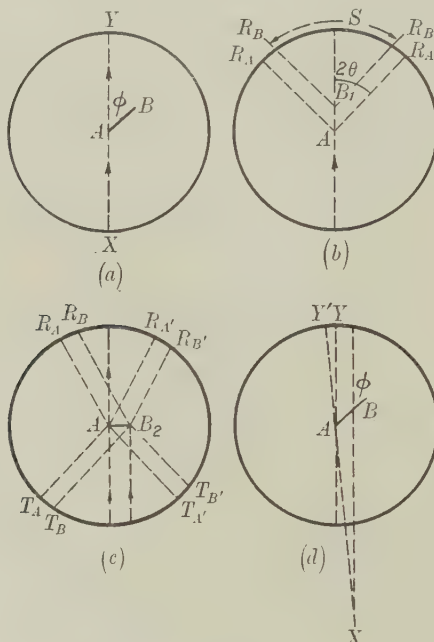$$= -2 \, (p \cos \phi) \sin 2\theta.$$

Fig. 3.

Hence

$$d\theta = \frac{\theta}{S} dS = -\frac{(p \cos \phi) \sin 2\theta}{2R} \qquad \ldots\ldots(11).$$

Combining equations (11) and (6)

$$da/a = (p \cos \phi/R) \cos^2 \theta \qquad \ldots\ldots(12).$$

A displacement of the specimen at right angles to the incident beam, figure 3 c, has the effect of crowding the lines together on one side of the film and spreading them out on the other side, but it has no effect on the value of S.

If the beam is divergent, figure 3 d, the same relations hold provided that ϕ is taken to mean the angle between the incident beam passing through B, and the direction of the displacement AB, i.e. $Y\hat{A}B$, not $Y'\hat{A}B$.

Equation (12) *therefore represents the relative error in lattice spacing introduced by a displacement of the specimen* by a distance p from the centre of the camera of radius R, while ϕ gives the direction of the displacement relative to the incident beam.

(ii) *Error due to absorption in the powder and divergence of the beam**

It is simplest to make the calculation for complete absorption. For partial absorption the effect is more complicated. In general it will result in the production of a line of varying intensity, the blackness diminishing from the outer edge of the line towards the inner edge. When the absorption coefficient is low it is convenient to measure to the centre of the line and make no correction. As the absorption coefficient increases the line becomes blacker on the outside and lighter on the inside. Finally, with very absorbent powders, only the outer portion of the line is visible, and measurements must be corrected.
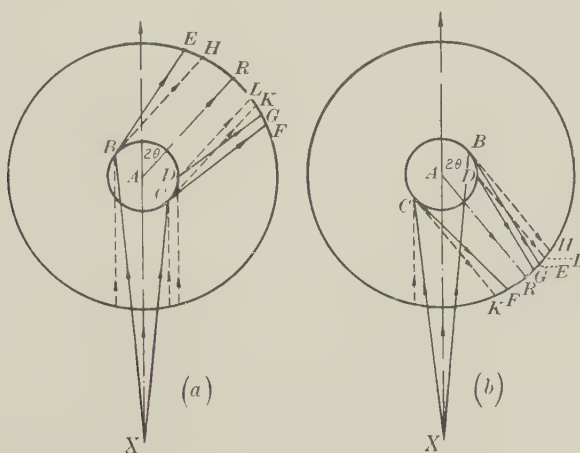

Fig. 4. Effect of absorption.

Figure 4 illustrates the effect of the error. Figure 4 (a) gives a small value of θ, figure 4 (b) a larger value. The specimen is immersed in a beam diverging from X. With a point specimen situated at A, the beam is deviated through an angle 2θ in the direction of AR, giving rise to a line on the film at R. With a specimen of finite size the ray AR is broadened to a beam, whose boundaries are defined by the rays BE and CF.

Owing to the effect of absorption, the intensity is not distributed symmetrically over the length EF, the greatest intensity being near the point F. If the substance is extremely absorbent the reflected beam is confined to a portion scattered from a narrow strip CD on the surface of the specimen, and the boundary rays are CF and DG. The beam strikes the film between F and G, and the point of measurement of the line will lie at some point between F and G, say at P. In order to estimate the error in the measured value of the lattice spacing, it is necessary to find the

* Compare O. Pauli, *Z.f. Krist.* **56**, 591 (1921); Hadding, *Centralblatt f. Min. etc.* p. 631 (1921). Hadding's expression for ds may be written $r (1 + \cos 2\theta)$.

distance RP. This is best done in two stages, a parallel beam being first considered and then the effect of divergence added.

A parallel incident beam would give rise to a reflected beam bounded by the rays BH and CK, where

$$HK = BC = 2r,$$

r being the radius of the specimen. Absorption limits the beam to the area bounded by the rays CK and DL, where $C\hat{A}D = 2\theta$, so that

$$RK = r, \quad RL = r \cos 2\theta.$$

The additional effect of divergence swings the rays CK and DL round to CF and DG respectively, where

$$G\hat{D}L = A\hat{X}D = r/AX,$$

$$F\hat{C}K = A\hat{X}C = (r/AX) \cos 2\theta.$$

Hence

$$LG = rR/AX, \quad KF = (rR/AX) \cos 2\theta.$$

The displacement of the outer edge of the line for a divergent beam RF can now be found. It is given by

$$RF = RK + KF = r\{1 + (R/AX) \cos 2\theta\} \qquad \ldots\ldots(13).$$

The displacement of the inner edge of the line RG is given by

$$RG = RL + LG = r(\cos 2\theta + R/AX) \qquad \ldots\ldots(14).$$

When a line is being measured it is most natural to choose a point somewhere near the centre of the line, but where the line is not symmetrical there is a bias towards the blacker side.

As a convenient approximation, we may suppose that measurement is made to the centre of gravity of blackening of the line. This is approximately at P, where

$$RP = \frac{r \sin 2\theta}{2\theta} \left(1 + \frac{R}{AX}\right) \qquad \ldots\ldots(15).$$

This is the most probable displacement of the measured position of the line. We deduce from this expression the following formulae for the errors involved in determining S, θ and a:

$$dS = 2RP = \frac{r \sin 2\theta}{\theta} \left(1 + \frac{R}{AX}\right),$$

$$d\theta = \frac{dS}{4R} = \frac{r \sin 2\theta}{4\theta} \left(\frac{1}{R} + \frac{1}{AX}\right) \qquad \ldots\ldots(16),$$

$$\frac{da}{a} = -\cot\theta \, d\theta = -\frac{r \cos^2 \theta}{2\theta} \left(\frac{1}{R} + \frac{1}{AX}\right) \qquad \ldots\ldots(17).$$

§4. GRAPHICAL TREATMENT OF ERRORS

On comparison of equations (12) and (17), it will be seen that the factor $\cos^2 \theta$ is common to both, so that the errors due to eccentricity and absorption are very similar in character. We may write a combined expression:

$$\frac{da}{a} = \left(\frac{p \cos \phi}{R} - \frac{r}{2\theta R} - \frac{r}{2\theta AX} \right) \cos^2 \theta \qquad \ldots\ldots(18).$$

If da be plotted against $\cos^2 \theta$ an approximately linear relationship will be obtained at large values of θ. The same will be true for the values of a deduced from the film measurement. In figure 5 the measured values of a are plotted for the corresponding values of $\cos^2 \theta$ between o and o·5. The line PQ is obtained. At P, $\cos^2 \theta = $ o, so that $da = $ o. P thus gives the true value of a.
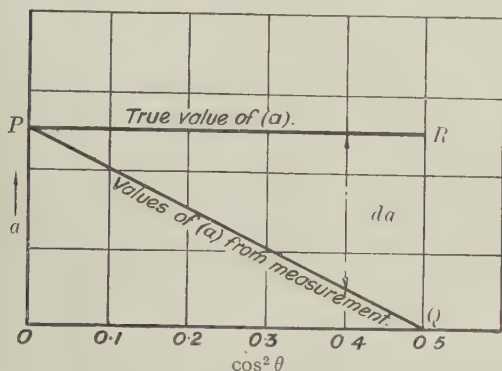

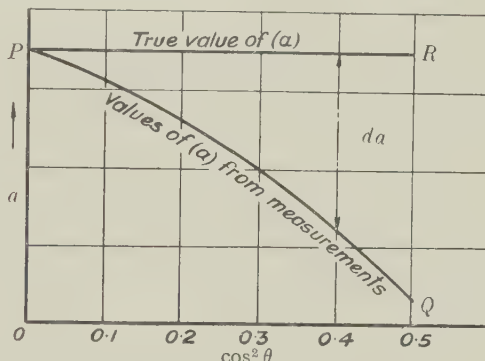Fig. 5 (*a*). Error curve for eccentricity.

Fig. 5 (*b*). Error curve for absorption.

The above process provides a means of ascertaining the correct value of a without further calculation. The degree of accuracy with which the point P can be found depends chiefly on whether there are a sufficient number of points on the line PQ in the neighbourhood of P. If there are lines only near the region where $\theta = $ o° and $\cos^2 \theta = $ 1, the position of P will be altogether uncertain. In practice a few lines in the neighbourhood of P serve to give its position with great accuracy, the lines nearer the centre of the film merely serving as pointers.

We therefore have the following rule for correcting the effects of eccentricity of specimen and absorption in the powder rod. *Plot the observed values of lattice spacing against the corresponding values of $\cos^2 \theta$. An approximately straight line should be obtained. Extrapolation to $\cos^2 \theta = $ o gives the correct value of the lattice spacing.*

§5. DISCUSSION AND EXAMPLES

Provided all necessary precautions are taken, this method of determining lattice spacings is accurate and reliable. The essential principles of the method are as follows:

(i) In order to calculate the glancing angle θ from the measurements S of the

film, we make use of a fixed angle θ_k, figure 1, corresponding to a given length of the film S_k, and apply equation (4) as follows:

$$\theta = \frac{\theta_k}{S_k} S.$$

By this means the uncertainty in measuring the radius is eliminated.

(ii) The values of the lattice spacing obtained from different lines on the film are plotted graphically against the corresponding values of $\cos^2 \theta$, and the true value of the lattice spacing is then obtained by an almost linear extrapolation to $\cos^2 \theta = 0$.

The fixed angle θ_k is found either by direct measurement of the camera, or by the use of a calibrating substance. The former method has the advantage that it involves no systematic error, but the latter is found to be more accurate provided that (a) the lattice spacing of the calibrating substance is very accurately known, and (b) there are no errors due to absorption in the powder specimen. Difficulties arise if the specimen is not exactly at the centre of the camera. We have recently found a means of overcoming these limitations by the use of quartz as the calibrating substance. A full account of this work will be published in a separate paper.

Using a camera of diameter 9 cm., it has been found possible to determine θ_k with an accuracy of about 1 part in 8000. This gives an accuracy of about 1 part in 30,000 in the final value of a determined by the extrapolation process. To test this point we have taken photographs of the same substance in three different cameras, calibrated in different ways. Details of this test are given in table 3.

Table 3. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 3. Cobalt K_α radiation

hkl	Radiation	S	θ	$\cos^2 \theta$	Lattice spacing
211	α	15·893	50·01	0·41292	2·8557
220	$\begin{cases}\alpha_1\\\alpha_2\end{cases}$	19·706 19·788	62·015 62·275	0·22015 0·21643	2·8588 2·8586
310	α_1	25·643	80·70	0·02610	2·8603
End of film	—	25·700 (S_k)	80·88 (θ_k)		
				Extrapolated value:	2·8605

Table 4. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 3. Cobalt K_α radiation. Thin specimen, diameter 0·3 mm.

hkl	Radiation	S	θ	$\cos^2 \theta$	Lattice spacing
211	α	15·830	49·92	0·41454	2·8595
220	$\begin{cases}\alpha_1\\\alpha_2\end{cases}$	19·648 19·727	61·96 62·205	0·22098 0·21744	2·8605 2·8603
310	$\begin{cases}\alpha_1\\\alpha_2\end{cases}$	25·583 25·835	80·675 81·47	0·02626 0·02200	2·8606 2·8606
End of film	—	26·164 (S_k)	82·51 (θ_k)		
				Extrapolated value:	2·8606

Table 5. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 3. Cobalt K_α radiation. Thick specimen, diameter 1·5 mm.

hkl	Radiation	S	θ	cos² θ	Lattice spacing
211	α	15·921	50·10	0·41145	2·852
220	α₁	19·725	62·075	0·21931	2·8575
	α₂	19·800	62·31	0·21594	2·8576
310	α₁	25·652	80·725	0·02597	2·8602
	α₂	25·905	81·52	0·02174	2·8602
End of film	—	26·220 (S_k)	82·51 (θ_k)		
				Extrapolated value:	2·8605

Table 6. Armco iron, remelted under a low pressure of hydrogen, filings annealed *in vacuo*. Camera 4. Cobalt K_α radiation

hkl	Radiation	S	θ	cos² θ	Lattice spacing
211	α	15·929	49·84	0·41590	2·8629
220	α₁	19·786	61·91	0·22171	2·8619
	α₂	19·862	62·145	0·21831	2·8619
310	α₁	25·783	80·675	0·02626	2·8606
	α₂	26·041	81·48	0·02197	2·8606
End of film	—	26·457 (S_k)	82·78 (θ_k)		
				Extrapolated value:	2·8604

Tables 3, 4, 5 and 6 show the results obtained from a specimen of Armco iron, remelted under a low pressure of hydrogen, the filings being annealed *in vacuo*. A photograph is given in figure 8. The absolute magnitude* of these values is not of special importance, since the iron is probably not absolutely pure, but the main point is that measurements made in different cameras give almost identical values for the lattice spacing on extrapolation to $\cos^2 \theta = 0$, in spite of differences in the size and centering of the powder rod. These differences give rise to considerable variation at large values of $\cos^2 \theta$, as may be seen from figure 6 where the values of the lattice spacing from tables 3 to 6 are plotted against the corresponding values of $\cos^2 \theta$. The curves are wide apart when $\cos^2 \theta$ is large, and come together as $\cos^2 \theta$ approaches zero. Curve 2 was obtained from a fairly thin specimen, taken in camera 2; the effects of absorption and eccentricity are, in this instance, additive. Curve 3 *a* was from a fairly thin specimen, taken in camera 3; absorption and centering effects almost cancel. Curve 3 *b*, from the same camera, shows the effect of using a very thick specimen. Curve 4 was obtained from a fairly thin specimen in camera 4; the eccentricity error here acts in the opposite direction to the absorption error, and in fact quite outweighs it.

* Some previous values found for iron are: 2·861 Å., A. Westgren, *J. Iron and Steel Inst.* **1**, 383 (1928). 2·8603 ± 0·0002 Å., F. C. Blake, *loc. cit.* 2·8600 ± 0·0005 Å., Preston, *Phil. Mag.* **13**, 419 (1932). 2·8607 Å., G. Phragmén, *J. Iron and Steel Inst.* **123**, 465.

The next examples show how the method may be used for the accurate comparison of specimens which differ only slightly in spacing. The materials compared were three specimens of nickel*: (i) a tiny specimen of electrolytic nickel as deposited on a copper wire; (ii) Mond nickel remelted under a low pressure of hydrogen and filings annealed *in vacuo*; (iii) as (ii), but comprising filings quenched from
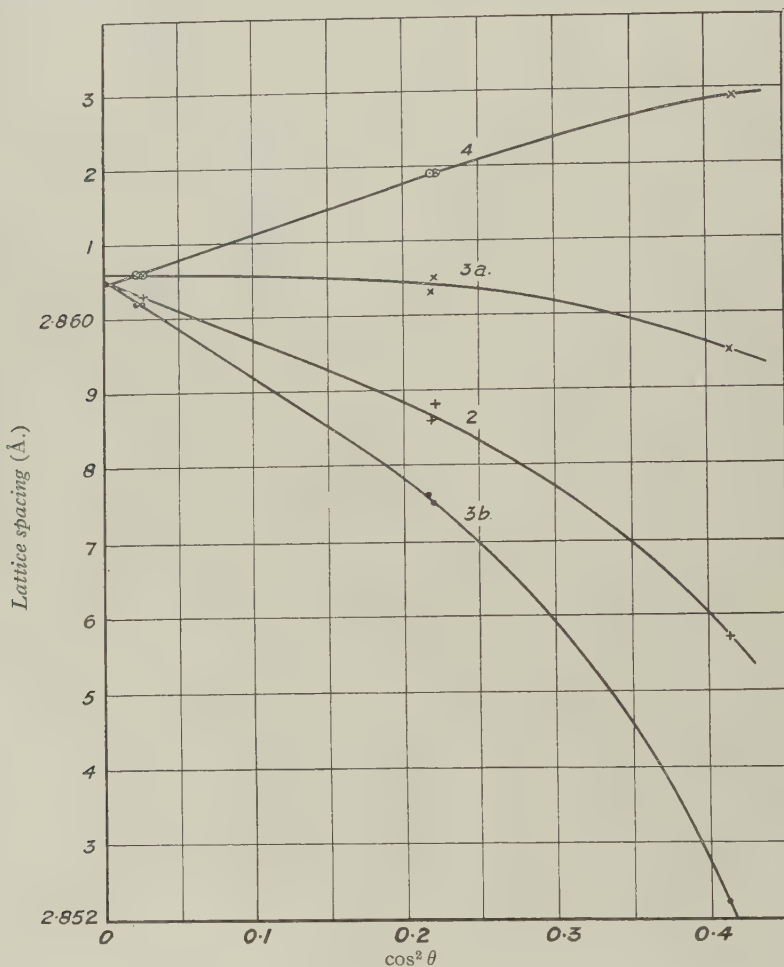

Fig. 6. Lattice spacing of iron-cobalt radiation.

700° C. after annealing. It will be seen that (ii) and (iii) give identical results, but the electrolytic nickel differs in spacing by an amount considerably greater than the experimental error. Photographs are given in figure 8.

Fig. 8. Powder photographs.

(1) Iron powder annealed, Cobalt K_α radiation.
(2) Nickel electro-deposited on copper wire, Copper K_α radiation.
(3) Nickel powder annealed, Copper K_α radiation.
(4) Nickel powder quenched 700° C., Copper K_α radiation.

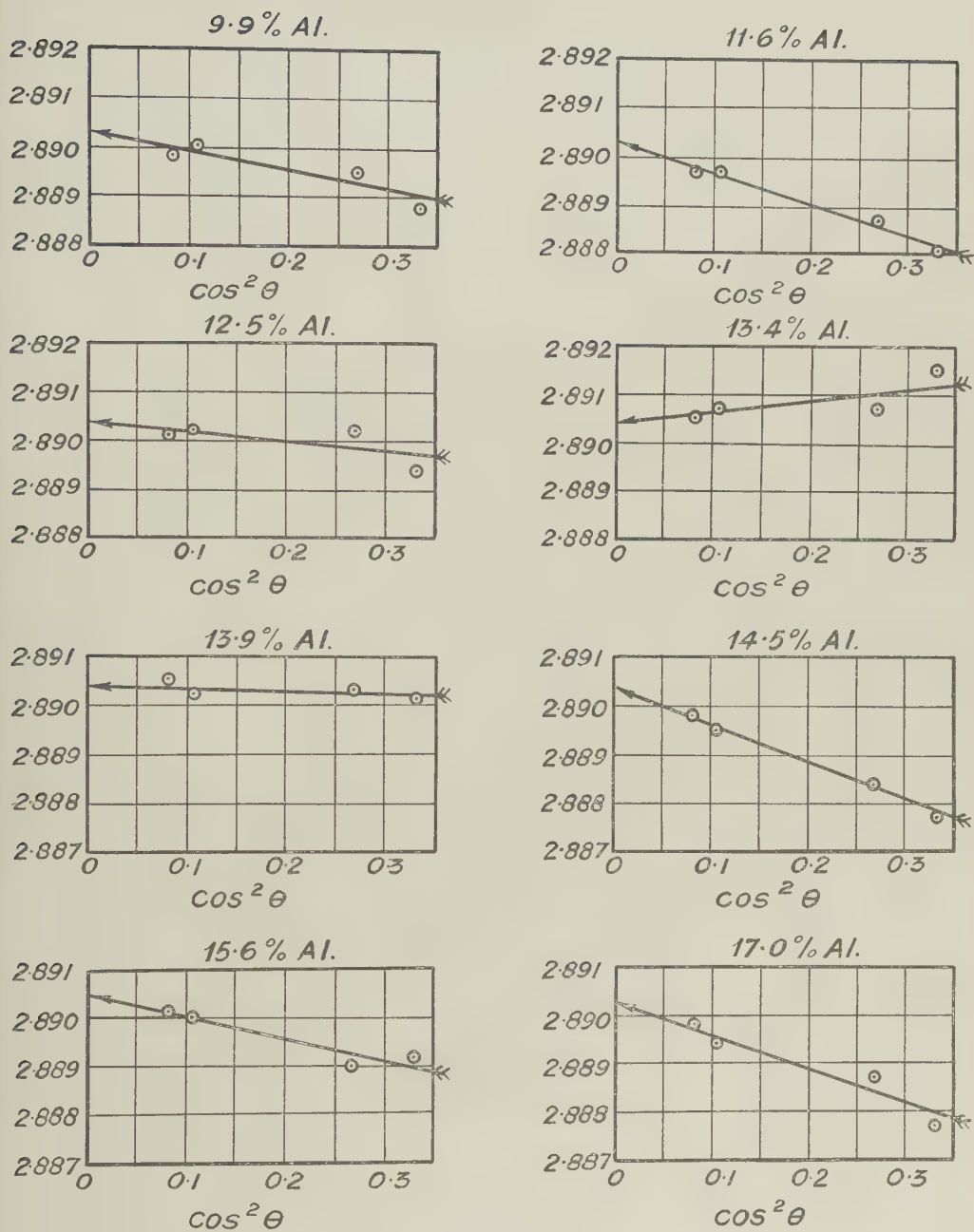Marks ↕ indicate the shadow of the knife-edge which acts as a fiducial mark giving S_k and θ_k.

Fig. 7. Lattice spacing of iron-aluminium alloys by extrapolation.

In table 7, which gives the results from the electro-deposited nickel, an interesting point arises. The α doublets were resolved right down to (311), but in spite of this they were never quite clear-cut, doubtless owing to the nickel being in a condition of strain. This has a somewhat peculiar effect on the measurements

Table 7. Nickel electro-deposited on a copper wire. Copper K_α radiation

Reflection	Σh^2	Radiation	S	θ	Lattice spacing	
400	16	α_1	19·334	60·98	3·5162	3·5160
		α_2	19·424	61·265	3·5154	
331	19	α_1	22·940	72·35	3·5163	3·5162
		α_2	23·090	72·825	3·5159	
420	20	α_1	24·684	77·85	3·5166	3·5162
		α_2	24·924	78·61	3·5155	
End of film	—	—	25·664 (S_k)	80·945 (θ_k)		
					Extrapolated value:	3·5162

Table 8. Nickel powder, annealed *in vacuo*. Copper K_α radiation

Reflection	Σh^2	Radiation	S	θ	Lattice spacing	
400	16	α_1	19·370	60·93	3·5178	3·5179
		α_2	19·452	61·19	3·5180	
331	19	α_1	22·982	72·29	3·5172	3·5173
		α_2	23·122	72·74	3·5174	
420	20	α_1	24·735	77·81	3·5169	3·5170
		α_2	24·953	78·49	3·5171	
End of film	—	—	25·731 (S_k)	80·945 (θ_k)		
					Extrapolated value:	3·5170

Table 9. Nickel powder, quenched from 700° C. Copper K_α radiation

Reflection	Σh^2	Radiation	S	θ	Lattice spacing	
400	16	α_1	19·362	60·94	3·5177	3·5176
		α_2	19·443	61·20	3·5176	
331	19	α_1	22·975	72·31	3·5170	3·5171
		α_2	23·117	72·75	3·5173	
420	20	α_1	24·718	77·80	3·5172	3·5172
		α_2	24·938	78·49	3·5171	
End of film	—	—	25·717 (S_k)	80·945 (θ_k)		
					Extrapolated value:	3·5170

and gives rise to an optical illusion. The α doublets in each case appear to be further apart than is actually the case. Thus the measurements from α_1 are not concordant with those from α_2. This is a phenomenon which we have observed many times, but it leads to no error if the curve for extrapolation is drawn between the α_1 and α_2 values. When this effect occurs, the values of the lattice spacing calculated from

measurement of α_1 lines are found to be greater than those from measurement of α_2 lines. Since α_1 is much stronger than α_2, the error in α_1 is likely to be less than the error in α_2, and so the curve for extrapolation is drawn nearer to α_1 than α_2. The last column of table 7 shows how the most probable value is obtained.

In conclusion, figure 7 shows the application of the extrapolation method to a series of annealed iron-aluminium alloys. It will be seen that within the range of compositions here given (9·9 per cent Al to 17 per cent Al) the lattice spacing is constant. This fact could not have been deduced except by extrapolation, since the individual measurements vary considerably from film to film owing to small differences, of the order of 0·1 mm., in the centering of the specimen.

§ 6. ACKNOWLEDGMENTS

IONIZATION CHARTS OF THE UPPER ATMOSPHERE

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ABSTRACT. In this paper Prof. Chapman's theory of the ionization of the upper atmosphere by solar radiation has been applied to construct a set of charts giving contour lines of equal ionic density over the surface of the earth. A simple approximate method of solving the fundamental differential equation of the theory by a rapid arithmetical process is described. Charts are drawn for winter, equinox and summer conditions for the values 0·5 and 1 of the parameter σ_0 and the value 150 of the parameter R. A brief comparison of these charts with existing empirical charts is given with a short discussion of its practical and theoretical significance.

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§ 1. INTRODUCTION

IT is well known that the propagation of wireless waves, especially of short waves below 100–200 metres, is intimately connected with the state of ionization of the upper atmosphere, and that this ionization goes through a well-defined diurnal cycle. It is therefore a matter of great importance in attempting to explain or to predict the behaviour of such waves to have a knowledge of the way in which the ionization varies with latitude and season and with local time, or, what comes to the same thing, to know at a given time the variation of ionic density over the surface of the earth at any given season.

The problem has been approached in a semi-empirical way by Eckersley and Tremellen*, who from a large amount of experimental data of skip distances, etc., and guided by general theoretical considerations, constructed a series of density charts for winter, equinox and summer conditions. These charts consist of a number of contour lines which divide them up into a series of grades A, $A–B$, B ... D, D' in order of decreasing ionic density, A being the intense daylight and D' the late night grade. They have proved very useful in studying the layer conditions for any given route and in explaining the observed variations with time and season of the average signal strengths received from stations in various parts of the world.

Now it would obviously be of great use if such charts could be constructed by purely theoretical methods since this procedure would not only help to elucidate uncertainties in the empirical charts but would also test the general soundness and adequacy of the theoretical ideas involved. Fortunately, a theory has been worked out in great detail by S. Chapman† on "the absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth," and the

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* T. L. Eckersley and K. Tremellen, *Proceedings of the World Engineering Congress, Tokyo*, **20**, 177 (1929).

† S. Chapman, *Proc. Phys. Soc.* **43**, 26, 483 (1931).

present paper represents the application of this theory to the above problem. Its chief purpose is to develop a general method of constructing such charts for any given values of the parameters involved in Chapman's theory, but in the practical illustrations given here values have been chosen to make the charts conform as nearly as possible to the empirical ones.

§ 2. THEORETICAL PRINCIPLES

Only a brief outline of the theory will be given here, as reference should be made to Chapman's paper for the detailed description of the notation and the development of the equations.

The rate of production I of ions per cm³, at a place in the upper atmosphere where the sun's zenithal angle is χ, is given by the expression

$$I/I_0 = \exp\left[1 - z - \exp\left(-z\right) . f\left(R, \chi\right)\right] = F\left(z, \chi\right) \quad ......(1),$$

where z is a linear function of the height, and $f\left(R, \chi\right)$ is a complicated function of a parameter R and the angle χ which is discussed in great detail in part 2 of Chapman's paper.

If α is the recombination coefficient of the ions, and the value of I were to remain constant at its maximum possible value I_0, the ionic density n would reach a maximum steady value n_0 given by

$$n_0 = \sqrt{(I_0/\alpha)} \qquad ......(2),$$

so that it is convenient to define a ratio ν such that

$$\nu = n/n_0 \qquad ......(3).$$

If the time is measured in terms of longitude ϕ expressed in radians, the differential equation for determining ν is

$$\sigma_0 . d\nu/d\phi + \nu^2 = F\left(z, \chi\right) \qquad ......(4),$$

where
$$1/\sigma_0 = 1.37 \times 10^4 \left(I_0 \alpha\right)^{\frac{1}{2}}$$
$$= 1.37 \times 10^4 \alpha n_0 \qquad ......(5).$$

σ_0 is thus another parameter which with R has to be chosen to fit the experimental data. χ for a given latitude and season is a function of ϕ, so that when the form of the function $F\left(z, \chi\right)$ is known equation (4) may be solved as a relation between ν and ϕ for a given value of z by approximate arithmetical methods.

This paper is partly concerned with the development of such a method which is accurate enough for the purpose in hand and at the same time adapted to rapid calculation. But before proceeding to this, there is a further theoretical point of great importance to be discussed. The contours on the charts may be interpreted as representing lines of equal maximum ionic density, and for any given point of the earth's surface the density represented is the maximum density in the layer at the time considered, since what really matters in the propagation of short wireless waves is the maximum density available and not the exact height at which it occurs. As is shown by Chapman's curves in figures 6–11, and more explicitly in figures

$12a$–$17b$ of part 1, this height varies with the local time ϕ. In order therefore to draw the contours correctly it would be necessary to solve equation (4) for a series of values of z and obtain a set of curves such as figure 6 of part 1. The envelope of these curves would be the required (ν, ϕ) curve. This process, however, would involve a great deal of time and labour in drawing a set of charts, and a simplifying assumption was made to reduce the work to reasonable dimensions.

An examination of figures 6–11 of part 1 shows that at least for $\sigma_0 = 1$ it is approximately true that the envelope is given by the curve corresponding to the height at which the rate of ionization is maximum at noon, and this assumption has been adopted in working out the charts. The approximation is probably near enough to give contours which are correct within the limits of experimental error, and in cases of doubt it should be remembered that the charts give a lower limit to the density, since the true envelope must lie above the chosen curve.

Now Chapman shows that for a given angle χ the height in terms of z at which the rate of ionization is greatest is given very nearly by $\log_e f(R, \chi)$, so that if z_0 is the height corresponding to χ_0, the value of χ at noon,

z_0
χ_0

$$z_0 = \log_e f(R, \chi_0) \qquad \ldots\ldots(6).$$

This is the value of z to be used in equation (4). Equation (1) now gives

$$F(z_0\chi) = \exp\left[1 - z_0 - \exp(-z_0).f(R, \chi)\right]$$
$$= \frac{1}{f(R, \chi_0)}.\exp\left[1 - \frac{f(R, \chi)}{f(R, \chi_0)}\right] \qquad \ldots\ldots(7).$$

§3. APPLICATION TO THE CONSTRUCTION OF THE CHARTS

The chief steps in the construction of the charts can now be formulated.

(a) The values of R and σ_0 must first be decided upon.

(b) The function $f(R, \chi)$ for the chosen value of R must be evaluated and plotted as a function of χ. This can be done with sufficient accuracy by interpolation from the values given in table 4 of part 2 of Chapman's paper.

(c) The function $F(z_0\chi)$ must be worked out from equation (7) and plotted as a function of χ for various values of χ_0, e.g. $\chi_0 = 0$, 15, 30, 45, 60, 75, 85 and 90°.

(d) A series of latitudes must be chosen, e.g. 0, 20, 40, 60 and 80°, and the differential equation solved for these latitudes for the seasons equinox ($\delta = 0$), winter ($\delta = -23\cdot5°$), and summer ($\delta = 23\cdot5°$). It is only necessary to work out the case of northern latitudes since the equinox chart will be symmetrical about the equator, and the southern hemisphere at the December solstice will be the same as the northern hemisphere at the June solstice and *vice versa*.

(e) In solving the differential equation for any given latitude and season, χ_0 is evaluated and the required values of $F(z_0\chi)$ are interpolated from the curves plotted in (c).

(f) A set of contour values must be decided upon (i.e. a set of ν values); from the (ν, ϕ) graphs obtained by solving the differential equation, the local times ϕ corresponding to a given ν for the various latitudes and seasons can be read off, and from the values obtained the charts can be constructed.

(g) By assigning values to α and n_0 consistent with the value of σ_0 chosen, the contours can be given absolute values of density (and hence of minimum transmissible wave-length).

§4. THE SOLUTION OF THE DIFFERENTIAL EQUATION

The equation to be solved is

$$\sigma_0 . dv/d\phi + v^2 = F \qquad \ldots\ldots(8),$$

where F is written for $F(z_0\chi)$.

Assume that the value of v at some time ϕ_1 is v_1 and is known, and that F_1 is the corresponding value of F. The value of v at a neighbouring time ϕ_2 may be expressed by means of Taylor's theorem as

$$v_2 = v_1 + (\phi_2 - \phi_1)\left(\frac{dv}{d\phi}\right)_1 + \frac{(\phi_2 - \phi_1)^2}{2}\left(\frac{d^2v}{d\phi^2}\right)_1 + \text{etc.,}$$

where $\left(\dfrac{dv}{d\phi}\right)_1$ is the value of $\dfrac{dv}{d\phi}$ at ϕ_1, etc. Assume that $(\phi_2 - \phi_1)$ is small and that

terms higher than the second order can be neglected, and further that

$$\left(\frac{d^2v}{d\phi^2}\right)_1 = \left[\left(\frac{dv}{d\phi}\right)_2 - \left(\frac{dv}{d\phi}\right)_1\right]\Big/ (\phi_2 - \phi_1),$$

so that

$$v_2 = v_1 + (\phi_2 - \phi_1).\frac{1}{2}\left[\left(\frac{dv}{d\phi}\right)_1 + \left(\frac{dv}{d\phi}\right)_2\right].$$

Geometrically this is equivalent to assuming that the slope of the chord joining the points is the mean of the slopes of the two tangents, and this is very nearly true provided that $(\phi_2 - \phi_1)$ is small and that there is no sharp bend in the curve between the points.

Now from equation (8)

$$\left(\frac{dv}{d\phi}\right)_1 = \frac{F_1 - v_1^2}{\sigma_0} \qquad \left(\frac{dv}{d\phi}\right)_2 = \frac{F_2 - v_2^2}{\sigma_0},$$

so that

$$v_2 = v_1 + \frac{(\phi_2 - \phi_1)}{2\sigma_0}[(F_1 + F_2) - (v_1^2 + v_2^2)].$$

Writing

$$(\phi_2 - \phi_1)/\sigma_0 = k \qquad \ldots\ldots(9),$$

this becomes

$$kv_2^2 + 2v_2 - \{2v_1 + k(F_1 + F_2) - kv_1^2\} = 0,$$

$$\therefore kv_2 = -1 + \sqrt{\{1 + 2kv_1 + k^2(F_1 + F_2) - k^2v_1^2\}}$$

$$= \sqrt{A} - 1 \qquad \ldots\ldots(10),$$

where

$$A = [2 + k^2(F_1 + F_2)] - (1 - kv_1)^2 \qquad \ldots\ldots(11).$$

v_2 is thus determined, and by a repeated application of the method the whole curve can be traced out.

For a given latitude and season a table is constructed with the values of ϕ decided upon (taking $\phi = 0$ at noon). The corresponding χ values having been worked out, the values of F are interpolated from the graphs described in (c) of

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§ 3. Adding them in adjacent pairs we obtain a column of $F_1 + F_2$ values from which $2 + k^2 (F_1 + F_2)$ can be tabulated.

Take as an example the case for the equinox and latitude 20° and $\sigma_0 = 1$. Then $\chi_0 = 20°$ and we have the results shown in table 1.

Table 1

ϕ (radians)	χ (degrees)	F	$F_1 + F_2$	k	$2 + k^2 (F_1 + F_2)$
0	20	0·94			
			1·83	0·3	2·1647
±0·3	26·1	0·89			
			1·65	0·3	2·1485
±0·6	39·2	0·76			
			1·28	0·3	2·1152
±0·9	54·2	0·52			
			0·71	0·3	2·0639
±1·2	70·5	0·19			
			0·24	0·15	2·0054
±1·35	78·1	0·047			
			0·047	0·15	2·0011
±1·5	86·1	0·000			

Now suppose, for example, it is assumed that $v_1 = 0·867$ when $\phi_1 = 0·6$, then we have the following scheme for determining the next four points:

$$\phi_2 = 0·9 \qquad k = 0·3 \qquad 2 + k^2 (F_1 + F_2) = 2·1152$$
$$\overline{kv_1 = 0·2601} \qquad (1 - kv_1) = 0·7399 \qquad \overline{(1 - kv_1)^2 = 0·5474}$$
$$\sqrt{A} = 1·2521 \qquad \qquad A = 1·5678$$
$$kv_2 = 0·2521 \qquad \underline{v_2 = 0·841}$$

$$\phi_2 = 1·2 \qquad k = 0·3 \qquad 2 + k^2 (F_1 + F_2) = 2·0639$$
$$\overline{kv_1 = 0·2521} \qquad (1 - kv_1) = 0·7479 \qquad \overline{(1 - kv_1)^2 = 0·5593}$$
$$\sqrt{A} = 1·2265 \qquad \qquad A = 1·5046$$
$$kv_2 = 0·2265 \qquad \underline{v_2 = 0·755}$$

$$\phi_2 = 1·35 \qquad k = 0·15 \qquad 2 + k^2 (F_1 + F_2) = 2·0054$$
$$\overline{kv_1 = 0·1133} \qquad (1 - kv_1) = 0·8867 \qquad \overline{(1 - kv_1)^2 = 0·7862}$$
$$\sqrt{A} = 1·1041 \qquad \qquad A = 1·2192$$
$$kv_2 = 0·1041 \qquad \underline{v_2 = 0·694}$$

$$\phi_2 = 1·5 \qquad k = 0·15 \qquad 2 + k^2 (F_1 + F_2) = 2·0011$$
$$\overline{kv_1 = 0·1041} \qquad (1 - kv_1) = 0·8959 \qquad \overline{(1 - kv_1)^2 = 0·8026}$$
$$\sqrt{A} = 1·0948 \qquad \qquad A = 1·1985$$
$$kv_2 = 0·0948 \qquad \underline{v_2 = 0·632}$$

The ease and rapidity of the method is obvious. The value of kv_1 is given directly by the previous value of kv_2, except where the value of $(\phi_2 - \phi_1)$ is altered, and by means of a set of four-figure tables of squares and a set of five-figure tables of square-roots the whole work is carried out with uniform accuracy.

There remains the problem of finding rapidly a correct value for the starting-point of these calculations. If v_s is the value of v at sunset (or more exactly when F becomes negligible), say ϕ_s, the decay of ionization throughout the night is represented by the equation

$$\frac{1}{v} - \frac{1}{v_s} = \frac{\phi - \phi_s}{\sigma_0} \qquad \dots\dots(12).$$

If then some value v_1 is assumed for v for a particular time ϕ_1 during the day, the above process can be used to trace the curve round to the sunset value v_s, and from equation (12) the value v_r to which the ionization has decayed by sunrise (i.e. when F again becomes appreciable) can be determined. With this value of v_r the above process can again be applied to trace the curve back to the original time ϕ_1, when the same value should be obtained if the correct value was chosen initially.

A little preliminary work shows that the maxima of the curves for a given value of σ_0 all occur at about the same time after noon except for the high latitudes, when it is noticeably later; and this value of ϕ can soon be found. At this value the relation $v = \sqrt{F}$ holds, and by choosing ϕ_1 as the nearest convenient value to the time of the maximum a near approximation can be made to the value of v_1. For a given change in the value of v_1 the change in the deduced v_s is much smaller, and the value of v_r is very insensitive to changes in v_s, except of course in the very short summer nights in high latitudes, so that an accurate value of v_r is in general obtained. The value of v_1 obtained with this value of v_r is often near enough to the original one, but if necessary the process can be continued to revise the original portion of the curve between v_1 and v_s, and the final curve is very near the true one and amply accurate enough for the construction of the charts.

§ 5. PRACTICAL ILLUSTRATIONS OF THE METHOD

(a) The empirical charts constructed to deal with the propagation of short waves really combine two effects, the effect of attenuation in the intense-daylight regions and the effect of electron limitation in the night regions. The first of these is mainly connected with the lower or E layer of the ionized regions and the second with the upper or F layer, and as the parameters R and σ_0 are quite different for these two layers it is impossible to represent the charts by a single set of values for R and σ_0. Some kind of composite chart would have to be constructed from two separate charts worked out with values of R and σ_0 corresponding to the E and F layers respectively.

Chapman has suggested for the E layer the values $R = 650$ and $\sigma_0 = 0.04$ and has taken these values as the basis of the detailed calculations involved in his paper. In the present paper attention is concentrated on the F layer for the study of the night grades, since they are perhaps the more important in the problem of long-distance transmissions.

For this layer the value of R will be taken as 150 for reasons which will be given in § 6, and charts will be constructed for $\sigma_0 = 1$ and $\sigma_0 = 0.5$.

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(*b*) In interpolating from table 4 of part 2 of Chapman's paper for the evaluation of $f(R, \chi)$ with R equal to 150, the value of $\log_e f(150, \chi)$ was taken as the average of the values of $\log_e f(100, \chi)$ and $\log_e f(200, \chi)$, as inspection of the table showed that this method of interpolation was very close and better than averaging the actual values of $f(100, \chi)$ and $f(200, \chi)$. The values of $\log_e f(150, \chi)$ and of $f(150, \chi)$ are given in table 2.

Table 2. Values of $\log_e f(150, \chi)$ and $f(150, \chi)$

χ	$\log_e f(150, \chi)$	$f(150, \chi)$	χ	$\log_e f(150, \chi)$	$f(150, \chi)$
30	0·1414	1·152	87	2·267	9·64
45	0·3394	1·404	90	2·705	15·00
60	0·6723	1·96	93	3·293	26·92
75	1·275	3·57	95	3·802	44·67
80	1·570	4·80	97	4·431	83·56
83	1·836	6·25	100	5·622	275·4
85	2·035	7·62			

z_0

$f(150, \chi)$ is plotted against χ in figure 1, and $\log_e f(150, \chi)$ is plotted against χ in figure 2, as by equation (6) this gives, for $\chi = \chi_0$, the value of z_0, the height at which the maximum ionization is assumed to occur.

(*c*) From equation (7) the values of $F(z_0 \chi)$ can be worked out for various values of χ_0 from the results of table 2. The values are given in table 3 and plotted in figure 3.

Table 3. Values of $F(z_0 \chi)$

χ	$\chi_0 = 0$	$\chi_0 = 15°$	$\chi_0 = 30°$	$\chi_0 = 45°$	$\chi_0 = 60°$	$\chi_0 = 75°$	$\chi_0 = 85°$	$\chi_0 = 90°$
0	1·000							
15	0·965	0·966						
30	0·859	0·863	0·868					
45	0·668	0·675	0·697	0·712				
60	0·383	0·396	0·427	0·479	0·510			
75	0·0765	0·0834	0·107	0·153	0·224	0·280		
80	0·0223	0·0254	0·0365	0·0635	0·120	0·199		
83	0·0053	0·0063	0·0103	0·0227	0·0571	0·132		
85	0·0013	0·0017	0·0032	0·0085	0·0284	0·090	0·131	
87	0·0002	0·0002	0·0005	0·0020	0·0102	0·0512	0·101	
90					0·0007	0·0114	0·0499	0·0667
93						0·0004	0·0105	0·0302
95							0·0010	0·0092
100								0·0002

(*d*) The latitudes suggested in § 3 were chosen, and table 4 gives the values of χ_0 and ϕ_s, where ϕ_s is the sunset time measured in radians from noon (so that $-\phi_s$ is sunrise time).

ϕ_s

(*e*) From the data of table 4 tables of the form of table 1 can be drawn up, the F values being interpolated from the curves of figure 3. The value of $(\phi_2 - \phi_1)$ has been made 0·3 in most cases except near sunrise, where the (ν, ϕ) curve bends rather sharply and the value 0·15 has been taken.
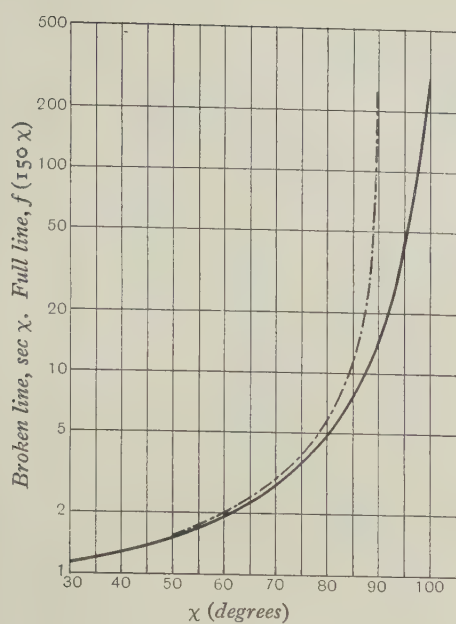
Fig. 1. $f(150, \chi)$ with $\sec \chi$ for comparison.
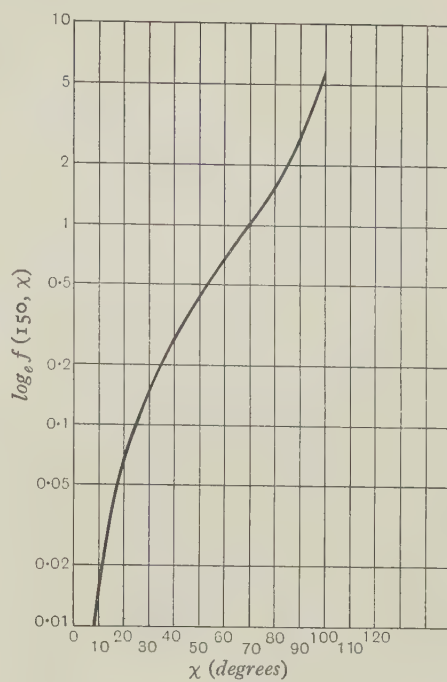
Fig. 2. $\log_e f(150, \chi)$.
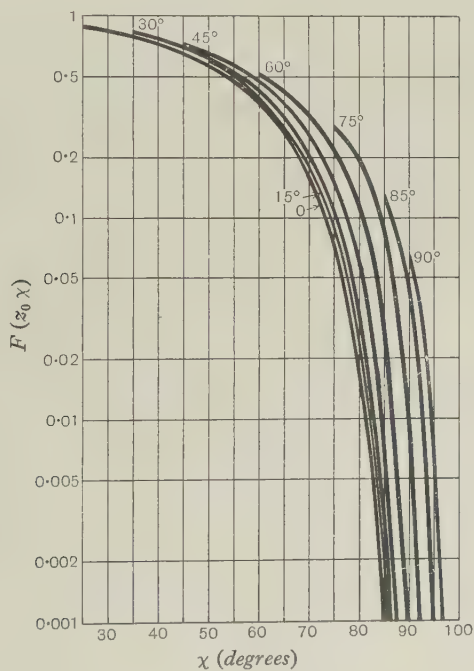
Fig. 3. $F(z_0 \chi)$; $\chi_0 = 0, 15, 30, 45, 60, 75, 85, 90°$.

The solution of the differential equation then follows along the lines of § 4. The (ν, ϕ) curves so obtained are reproduced in figures 4–9. Figures 4–6 are for the

Table 4. Values of χ_0 and ϕ_s

Latitude (°)	Equinox $\delta = 0$		Winter $\delta = -23\cdot5°$		Summer $\delta = 23\cdot5°$	
	χ_0 (°)	ϕ_s	χ_0 (°)	ϕ_s	χ_0 (°)	ϕ_s
0	0	1·57	23·5	1·57	23·5	1·57
20	20	1·57	43·5	1·41	3·5	1·73
40	40	1·57	63·5	1·20	16·5	1·95
60	60	1·57	83·5	0·719	36·5	2·42
80	80	1·57			56·5	
90					66·5	
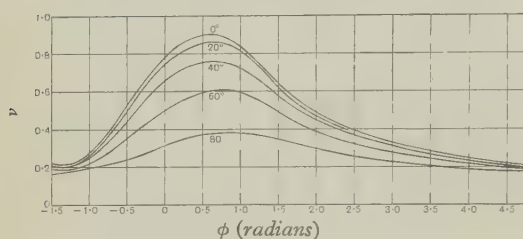
Fig. 4. Equinox: $\sigma_0 = 1$. Latitudes 0, 20, 40, 60, 80°.
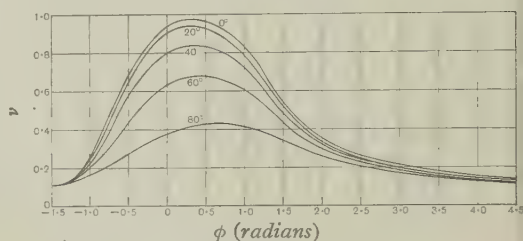
Fig. 7. Equinox: $\sigma_0 = 0\cdot5$. Latitudes 0, 20, 40, 60, 80°.
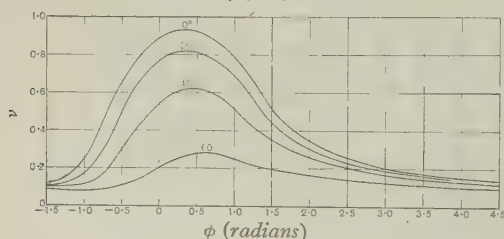
Fig. 5. Winter: $\sigma_0 = 1$. Latitudes 0, 20, 40, 60°.
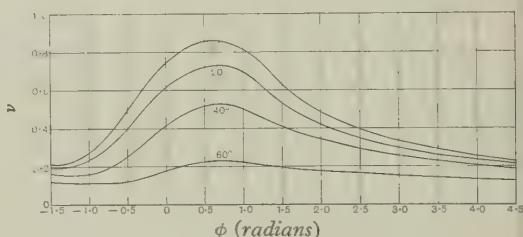
Fig. 8. Winter: $\sigma_0 = 0\cdot5$. Latitudes 0, 20, 40, 60°.
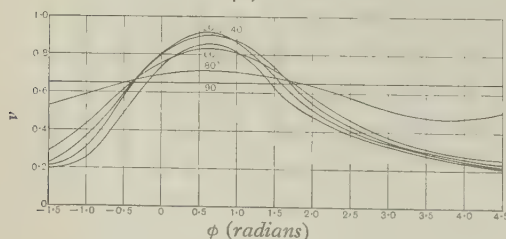
Fig. 6. Summer: $\sigma_0 = 1$. Latitudes 0, 20, 40, 60, 80, 90°.
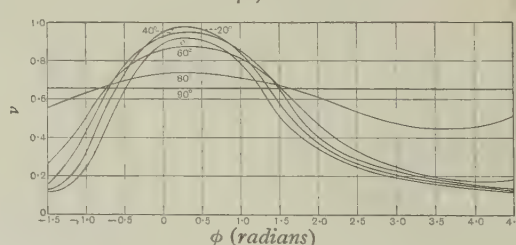
Fig. 9. Summer: $\sigma_0 = 0\cdot5$. Latitudes 0, 20, 40, 60, 80, 90°.

value $\sigma_0 = 1$ for equinox, winter, and summer respectively, and figures 7–9 and the corresponding curves for $\sigma_0 = 0\cdot5$. These curves show up well the approximately constant position of the maximum ionization with the gradual shift towards

sunset for high latitudes, and the smaller shift with greater maximum ionization and the more rapid decay for the smaller value of σ_0.

(f) The values of ν decided upon for the construction of the charts were 0·75, 0·65, 0·55, 0·45, 0·35, 0·30, 0·25, 0·20, 0·15 and 0·10. From the (ν, ϕ) curves in figures 4–9 tables were made of the ϕ values corresponding to these ν values*. These ϕ values were then converted to longitudes from which the charts could be constructed. In keeping with the empirical charts they have been drawn on a Mercator's projection with a G.m.t. scale and the scale was adjusted for use with the same map
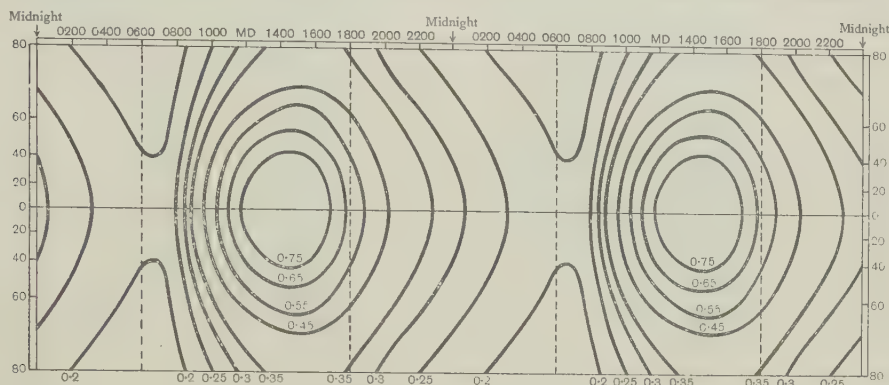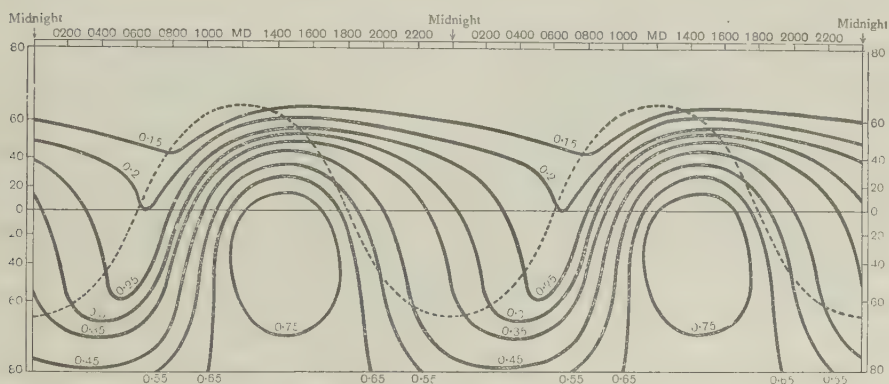

Fig. 10. Equinox chart. $\sigma_0 = 1$.

Fig. 11. Winter chart. $\sigma_0 = 1$.

as for the empirical charts. The charts are reproduced on a reduced scale in figures 10–13. The summer charts are not shown as they are simply mirror images of the winter charts†.

(g) For an assumed value of n_0 the absolute value of n (or νn_0) is known, and for a given density the minimum wave-length that can be transmitted (i.e. that does not escape through the layer) is known when the height of the position of maximum density is known. If R_0 is the radius of the earth and h the height above the surface R_0, h

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* For the region between sunset and sunrise it was found best to use a set of linear graphs of ν^{-1} against ϕ; see equation (12).

† The broken lines are the sunrise-sunset great-circles.

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of the earth to which the limiting wave-length λ_{min} penetrates, then from trans-mission theory

$$\lambda_{min} = \lambda_0 \sqrt{\{h\,(2R_0 + h)\}/(R_0 + h)} \qquad \ldots\ldots(13),$$

where λ_0 is the critical wave-length corresponding to the density n and is given by

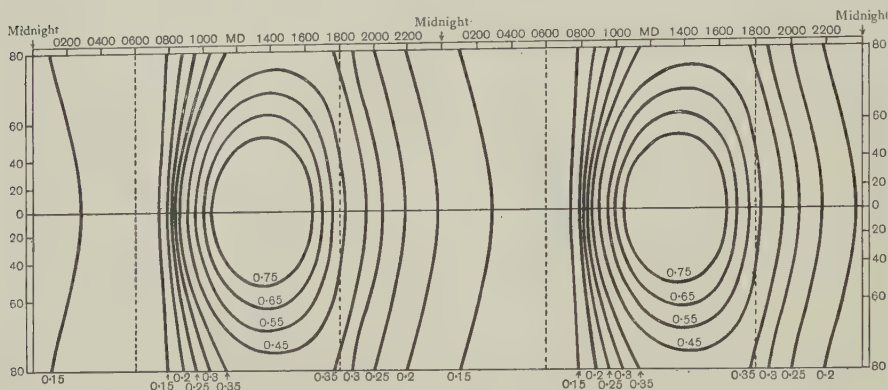$$\lambda_0{}^2 = 1\cdot117 \times 10^9/n \text{ metres}^2 \qquad \ldots\ldots(14).$$

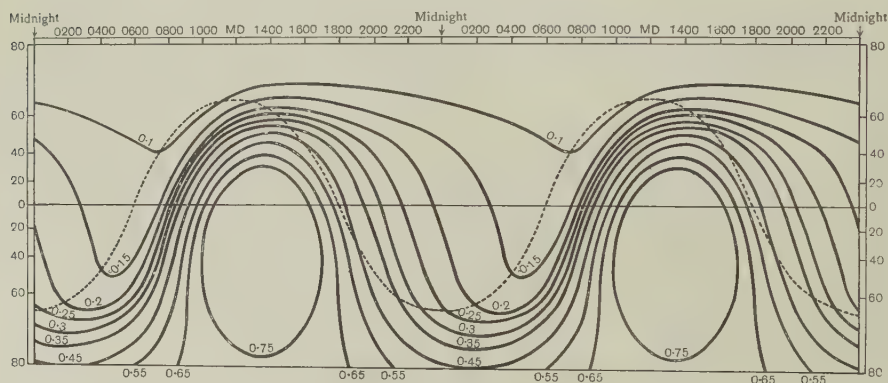Fig. 12. Equinox chart. $\sigma_0 = 0\cdot5$.

Fig. 13. Winter chart. $\sigma_0 = 0\cdot5$.

Taking R_0 equal to 6366 km. and h to 300 km., for the upper layer (13) becomes

$$\lambda_{min} = \lambda_0.0\cdot297 \qquad \ldots\ldots(15).$$

If equation (13) is written $\lambda_{min} = p\lambda_0$, for a given density, and therefore from (14) for a given critical wave-length λ_0, λ_{min} is a function of p, i.e. of h. Taking h_0 equal to 300 km. as datum, in the general case h may be written as $300 + z_0 H$, and if p_0 is the value of p when $h_0 = 300$ km. and $(\lambda_{min})_0$ the value of λ_{min} for p_0

$$\lambda_{min} = (\lambda_{min})_0\,p/p_0 \qquad \ldots\ldots(16),$$

p/p_0 is a function of z_0 and is plotted in figure 14. This relation is of some importance in assigning minimum wave-length values to the contour lines, since in higher

latitudes in winter z_0 is of the order of 2, and the minimum wave-length may be increased by more than 10 per cent over the equator value for the same contour.

The limits of the value of σ_0, 0·5 to 1·0, are fixed by the facsimile measurements* of the coefficient of recombination and of the ionic density. The values of α and n_0 are not very accurately known, and in adjusting them to satisfy equation (5) α may be taken between the limits of 7×10^{-11} and 10^{-10}, giving n between 10^6 and 7×10^5 approximately for $\sigma_0 = 1$, or between 2×10^6 and $1·4 \times 10^6$ for $\sigma_0 = 0·5$. In attempting to make the charts agree with the empirical ones, n_0 can be adjusted to give the best fit within these limits.
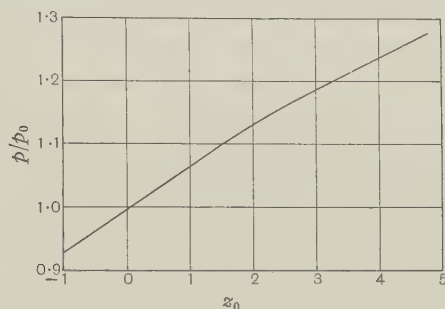

Fig. 14. p/p_0 when $h_0 = 300$ km. and $R = 150$.

§6. DISCUSSION OF RESULTS

The value of the parameter R in Chapman's theory has been made 150. R depends upon H, and H itself is only very imperfectly known since it depends upon the constitution of the upper atmosphere in a region concerning which the physical data are very scanty. H depends upon the temperature T of the atmosphere and upon the mean atomic or molecular mass m, being given by $H = kT/mg$, where k is Boltzmann's constant $1·37 \times 10^{-16}$, and g is the acceleration due to gravity, 981 cm./sec.² If M is the mean atomic or molecular weight

$$m = M \times \text{(mass of the hydrogen atom)}$$
$$= 1·66 \times 10^{-24} \times M \text{ gm.,}$$

and
$$H = 1·37 \times 10^{-16} \times T/1·66 \times 10^{-24} \times 981 \times M \text{ cm.}$$
$$= 0·84 \ T/M \text{ km.}$$

The value of T is very uncertain at the height of the upper layer, a value of 300° K. being possible and of 1000° K. not impossible, while M depends upon the state of dissociation and ionization of the atmosphere. Assuming with Chapman that the ionization is due to the absorption of wave-lengths below 910 Å. by atomic oxygen, an idea of the possible limits of H can be obtained by taking T equal to 300° K. and M to 16 for ionized atomic oxygen, and T equal to 1000° K. and M to 8 for wholly ionized atomic oxygen, giving $H = 15·7$ km. and 105 km. respectively. It was felt that it would be best to take a value of about 50 km., and since the cal-

* T. L. Eckersley, *J.I.E.E.*

culations in the earlier part of the paper are not greatly affected by a moderate change in the value of the parameter R, R was actually taken as $45 \cdot 1$ km. as this makes R equal to 150 if h_0 is 300 km., which is convenient for the easy interpolation of $f(R, \chi)$ as described in paragraph (*b*) of § 5.

The selection of the two values of σ_0, $0 \cdot 5$ and $1 \cdot 0$, is to take account of the fact established by facsimile experiments and by signal-measurements and commercial traffic experience that the general level of ionization is now markedly less than it was three years ago. This effect seems to be connected with the sunspot cycle and to be more marked in the higher latitudes than in the tropics. The lower value of σ_0 may be regarded as a sunspot-maximum value, and the upper one as a sunspot-minimum value. Facsimile results show that the recombination coefficient does not alter within the limits 7×10^{-11} and 10^{-10} given above, so that the value of σ_0 must be varied inversely as the general level of ionization.

The theoretical charts show a good general agreement with the empirical ones. By a suitable adjustment of n_0 within the limits set out in paragraph (*g*) of § 5, the agreement can be made quite good for the summer chart, especially in the tropics, and gives weight to the general soundness of the theoretical principles involved. In the northern latitudes, however, there is a definite discrepancy between the charts, especially for winter. In the region of noon for $\sigma_0 = 1$ the less dense grades extend too far towards the equator, while if σ_0 is altered to $0 \cdot 5$ to increase the general level of ionization in the noon region, the more rapid decay during the night makes the late night regions too thin. These discrepancies are apart from those arising from the attenuating effect of the lower layer, and they lead to the conclusion that they must be due to some additional cause not taken into account in the theory. Some extra source of ionization which will follow the sunspot cycle is needed in these latitudes. Chapman has outlined a theory of magnetic storms suggesting the projection from the sun of a neutral ionized stream of particles, which, when it gets within a distance from the earth equal to the earth's diameter, begins to get separated out by the earth's magnetic field, the electrons being deflected towards the polar regions. Such a theory throws light on the problems of magnetic storms and aurorae, and indicates an explanation of scattering. If an estimate could be made of the magnitude and diurnal variation of the rate of ionization, this effect could be included in the differential equation by the necessary modification of the F function. That the effect must be diurnal is seen by working out the case of a constant source of ionization. Figures 6 and 9 show the amount of ionization at the north pole in the summer due to quite a small constant rate of ionization. Owing to the comparatively slow rate of recombination the steady value of the ionization is quite large, and similarly quite a small additional constant ionization will have the effect of flattening the (ν, ϕ) curves considerably and of destroying the contrast between day and night regions which actually exists in the higher latitudes.

§7. CONCLUSION

As was stated in the introduction, the main object of this paper has been to develop a technique for the construction of charts from Chapman's theory, but the short discussion given above shows the usefulness of such charts, and indicates the main conclusion obtained from comparison with the empirical charts. It has certainly helped to bring out the general properties of such charts by showing more clearly the effect of, say, a larger recombination coefficient or an increase in the maximum ionic density, and it has in turn suggested possible ways of modifying the theory to fit all the experimental facts.

§8. ACKNOWLEDGMENTS

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FURTHER INVESTIGATION OF THE ARC SPECTRUM OF ARSENIC

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Communicated by Prof. A. Fowler, F.R.S., June 2, 1932. Read July 8, 1932.

ABSTRACT. By photographing the spectrum of arsenic by the method of the hollow-cathode discharge in helium and in neon about 100 new lines have been recorded. In the light of the experimental data, the analysis of As I published by previous investigators has been considerably altered and extended. Several new levels have been added and the higher members of the chief groups of the ms series of terms have been identified. A mean value of 85,000 cm.$^{-1}$ has been suggested for the deepest term $4p\ {}^4S_2$ which leads to a first ionization potential of approximately 10·5 V. for arsenic.

§ 1. INTRODUCTION

IN spite of numerous investigations of the spectra of the elements of the fifth group of the periodic table, our knowledge of the arc spectra of these elements remains incomplete; it is only in the case of N I that the absolute values of the characteristic energy levels are known with a fair degree of accuracy. Phosphorus presents many experimental difficulties and the spectrum itself is not known completely, while arsenic, antimony and bismuth, although the excitation of their spectra is easy, contain several important lines in spectral regions which are difficult to explore.

In a recent paper*, K. R. Rao has described experiments on the arc spectrum of arsenic, particularly in the region λ 2200 to λ 1563, which have led to the identification of the combinations between the deepest set of terms $4p\ {}^4S$, ${}^2D$ and ${}^2P$ of As I and the higher $5s$ and $4p$ terms. Combinations with some of the terms of the $4d$ state have also been suggested. The interesting feature of his analysis is the location of the $4p'$ terms, chiefly $4p'\ {}^4P$, which involve the displacement of an inner s electron.

Later, Meggers and de Bruin† have independently investigated this spectrum in the region λ 10024 to λ 1890 and have suggested a similar scheme. The most important contribution towards the analysis of As I in the work of these authors is the identification, though partially, of the multiplets in the infra-red, arising from the addition of a $5p$ electron to the s^2p^2 configuration of the core. However, in extending the analysis into the Schumann region, they have utilized the somewhat incomplete data published by L. and E. Bloch‡. For a complete bibliography of the previous work on the arc spectrum of arsenic, reference must be made to these two papers.

There is a large measure of agreement between the schemes published by K. R. Rao and by Meggers and de Bruin. Their conclusions regarding the fundamental multiplets resulting from the combination of the $4p$ terms with the $5s\ {}^4P$, ${}^2P$ and ${}^2D$ are identical. Of the 30 levels assigned to the $4p$, $5s$, or $4d$ states by Meggers and
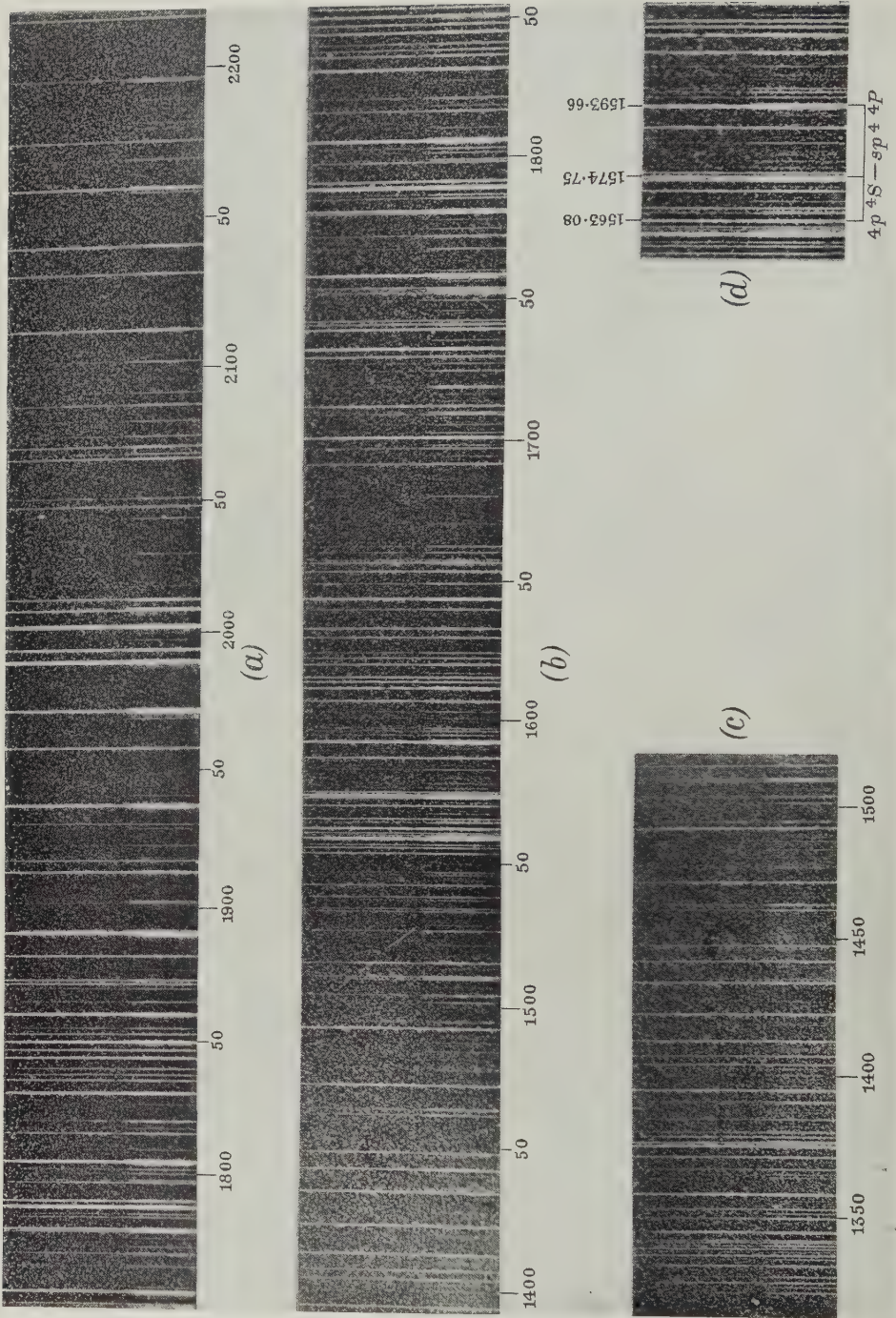
* *Proc. R. S. A*, **125**, 238 (1929).   † *Bur. Standards J. Res.* **3**, 765 (1929).

‡ *J. Phys.* **4**, 622 (1914); and *Comptes Rendus*, **171**, 709 (1920).

(a), (b), (c) Spectrum of arsenic obtained with the hollow-cathode discharge in helium in the region $\lambda 2200$ to $\lambda 1300$.

(d) Portion of the same showing the group $4p\,^4S - sp^4\,^4P$.

de Bruin and 35 assigned by Rao, there are as many as 25 common levels. The chief point of disagreement lies in the designation of these levels, particularly of those ascribed to the $4d$ state; there is, in some cases, a difference in the assignments even of the inner quantum number to the level. It must be stated at the outset that, as remarked both by Rao and by Meggers and de Bruin, it is difficult to define with certainty the $4d$ group of terms, as there are no means of verifying the correctness of the location either by other combinations such as $4d \leftarrow 5p$ or by comparison with other spectra.

In the present communication, these two schemes are discussed in detail and considerably altered and extended in the light of further experimental work. Many additional energy levels of As I have been discovered, as a result of the extension of the spectrum further into the ultra-violet. The higher members of the chief groups have been found and a probable estimate made of the absolute values of the characteristic terms of As I.

§ 2. EXPERIMENTAL

All the previous investigators have utilized the ordinary arc in air between metallic arsenic contained in electrodes of carbon, aluminium or copper. In his investigation of the Schumann region, Rao had recourse to the special method* designed for Prof. Fowler's work on silicon, which uses essentially an arc in an atmosphere of nitrogen.

The experimental work which formed the basis of the present scheme was carried out by Dr K. R. Rao and Dr J. S. Badami in Prof. Paschen's laboratory at the Physikalisch-Technische Reichsanstalt. The author is greatly indebted to them for placing their plates at his disposal. The mode of excitation of the arc spectrum was the hollow cathode discharge in helium or neon. The details of its use have been previously described by various writers†. In the present experiments the hollow cathode was of carbon or tungsten; the latter was found better. To start with, the system was exhausted to a hard vacuum and helium or neon was then kept in circulation by a mercury vapour pump and purified by passing over hot copper oxide and over charcoal cooled by liquid air. A constant potential difference of about 800 V. was applied to the electrodes and a steady current of 200 to 300 mA. was maintained throughout the exposure. The bright negative glow filling the inside of the hollow cathode was photographed by a vacuum spectrograph of radius 1 m. and giving a dispersion of about 17 Å./mm. Exposures varying from six to eight hours were found necessary. The entire region of the spectrum between λ 2800 and λ 500 was obtained at a single exposure. The usual lines of He, Ne, C were used as standards in reducing the plates. It is thought that the wave-lengths are accurate to about 0·03 Å.

§ 3. STRUCTURE OF THE SPECTRUM

The characteristic terms of the arc spectrum of As are built up, according to Hund's theory, on the three terms ^3P, ^1D, ^1S, of the s^2p^2 configuration of As⁺. A further group of terms is based on the state sp^3 of the core. The most important of

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* A. Fowler, *Proc. R. S.* A, 123, 422 (1929). E. W. H. Selwyn, *Proc. Phys. Soc.* 41, 392 (1929).

† F. Paschen, *Sitzungsb. d. Preus. Akad. d. Wiss.* p. 207 (1927). R. Frerichs, *Ann. d. Phys.* 85, 362 (1928).

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these predicted terms are set out in table 1. To avoid the use of fractions, an integral inner quantum number, greater by half a unit than the true value, has been assigned to each term throughout this paper. The notation is in accordance with that used in the previous paper on As I*.

Table 1. Theoretical terms of As I

3_1 3_2 3_3	4_1 4_2 4_3 4_4	5_1 5_2 5_3	6_1	Term prefix	Terms Limit ^3P	^1D	^1S
2 6 10	2 3 . .	. . . .	.	$4p$	^4S	^2D	^2P
2 6 10	2 2 . .	I . .	.	$5s$	$^4P\ ^2P$	^2D	^2S
2 6 10	2 2 I .	. . .	.	$4d$	$^4F\ ^4D\ ^4P$ / $^2F\ ^2D\ ^2P$	$^2G\ ^2F^a\ ^2D^a$ / $^2P^a\ ^2S^a$	$^2D^b$
. . .	. . . .	. . .	.				
2 6 10	2 2 . .	. I .	.	$5p$	$^4D\ ^4P\ ^4S$ / $^2D\ ^2P\ ^2S$	$^2F\ ^2D^a$ / $^2P^a$	$^2P^b$
. . .	. . . .	. . .	.				
2 6 10	2 2 . .	. . .	I	$6s$	$^4P\ ^2P$	^2D	^2S
2 6 10	I 4 . .	. . .	.	sp^4	$^4P\ ^2P$	^2D	^2S

The principal multiplets of As I which contain most of the prominent lines are presented in table 2, in which the classifications made by K. R. Rao and by Meggers and de Bruin have also been indicated in the first two columns for purposes of comparison.

Table 2. Multiplets in As I

M & B	R	$4p$......	$^4S_2$ 85000	$^2D_2$ 74409·7	$^2D_3$ 74087·4	$^2P_1$ 66816·7	$^2P_2$ 66355·5
				10590·3	322·3	7270·7	461·16
$5s\ ^4P_1$	$5s\ ^4P_1$	$5s\ ^4P_1 = 34308{\cdot}0$ *915·8*	50693·5 (10)	40102·0 (5)	—	32507·56 (2)	32046·38 (
$^4P_2$	$^4P_2$	$^4P_2 = 33392{\cdot}2$ *1287·7*	51608·1 (12)	41018·0 (4)	40695·6 (5)	33423·9 (2)	32962·77 (
$^4P_3$	$^4P_3$	$^4P_3 = 32104{\cdot}5$	52895·2 (15)	42305·4 (3)	41983·0 (6)	—	34250·3 (
$^2P_1$	$^2P_1$	$^2P_1 = 31866{\cdot}8$ *1469·6*	53134·1 (4)	42543·2 (9)	—	34949·27 (7)	34488·12 (
$^2P_2$	$^2P_2$	$^2P_2 = 30397{\cdot}2$	54603·3 (6)	44012·4 (3)	43690·5 (10)	36419·24 (6)	35958·09 (
$^2D_2$	$^2D_2$	$^2D_2 = 24167{\cdot}8$ *−19·1*	60833·2 (5)	50241·9 (9)	49920·6 (4)	42648·6 (5)	42187·1 (
$^2D_3$	$^2D_3$	$^2D_3 = 24186{\cdot}9$	60812·5 (5)	50224·0 (7)	49901·4 (10)	—	42167·0 (
$4d\ ^4P_1$	$5s\ ^2S_1$	$^2S_1 = 19505{\cdot}2$	65492·2 (3)	54901·6 (4)	—	47311·3 (5)	46850·6 (
$4d\ ^4P_3$	$sp^4\ ^2P_2$	$6s\ ^4P_1 = 17995{\cdot}4$ *912·2*	67003·9 (6)	56413·3 (2)	—	48822·4 (2)	48360·7 (
$4d\ ^2P_1$	a	$^4P_2 = 17083{\cdot}2$ *1392·1*	67915·9 (7)	—	57004·7 (3)	49733·8 (3)	49272·6 (
—	—	$^4P_3 = 15691{\cdot}1$	69399·2 (8)	58718·2 (1)	58397·2 (2)	—	50663·7 (
$4d\ ^2P_2$	$sp^4\ ^2P_1$	$^2P_1 = 16689{\cdot}0$ *1382·5*	68310·1 (5)	57719·7 (6)	—	50128·6 (5)	49667·4 (
—	β	$^2P_2 = 15306{\cdot}5$	69695·1 (8)	59105·5 (5)	58781·4 (8)	51506·0 (2)	51048·8 (

* K. R. Rao, *loc. cit.*

Table 2. Multiplets in As I (continued)

M & B	R	4p......	$^4S_2$ 85000	$^2D_2$ 74409·7	$^2D_3$ 74087·4	$^2P_1$ 66816·7	$^2P_2$ 66355·5
			10590·3		322·3	7270·7	461·16
4d $^4D_1$	sp^4 $^4P_1$	sp^4 $^4P_1$=21022·5 −477·3	63976·3 (7)	53387·7 (4)	—	45794·6 (2)	45333·2 (2)
4d $^4F_2$	$^4P_2$	$^4P_2$=21499·8 −752·1	63502·2 (8)	[52910]	52587·6 (2)	45315·9 (2)	44854·8 (3)
4d $^4F_3$	$^4P_3$	$^4P_3$=22251·9	62748·6 (9)	52158·3 (3)	51835·7 (1)	—	44102·4 (2)
—	4d $^2P_2$	$^2P_2$=12224·2	72784·4 (4)	62182·4 (6)	61859·8 (5)	54591·1 (5)	54130·4 (5)
—	4d $^2P_1$	$^2D_2$=12486·7 89·0	72513·7 (4)	61922·6 (5)	61599·9 (4)	54330·7 (4)	—
—	—	$^2D_3$=12397·7	72602·1 (3)	62012·0 (7)	61689·6 (6)	—	53958·1 (4)
—	δ	$^2S_1$=11371·9	—	—	—	55444·7 (2)	54983·7 (2)
5s $^2S_1$	sp^4 $^2D_{2,3}$	4d $^4F_2$=20192·9 −468·2	64808·4 (4)	54215·5 (5)	53894·4 (3)	46624·0 (5)	46162·6 (6)
4d $^4D_3$	4d $^2F_3$	$^4F_3$=20661·1 618·4	64337·6 (4)	53749·6 (6)	53426·2 (4)	—	45694·7 (1)
—	—	$^4F_4$=20042·7	—	—	54044·7 (6)	—	—
—	—	$^2F_3$=19045·2 1820·6	—	55364·5 (9)	55042·1 (4)	—	—
—	—	$^2F_4$=17224·6	—	—	56862·8 (9)	—	—
4d $^4P_2$	sp^4 $^2S_1$	4d $^4D_1$=18416·7 −102·3	66583·7 (3)	55992·0 (3)	—	48400·8 (3)	47938·7 (3)
4d $^4P_1$	4d $^4D_2$	$^4D_2$=18519·0 239·5	66479·7 (5)	55889·4 (7)	55567·6 (3)	48298·6 (3)	47838·9 (2)
—	4d $^4D_3$	$^4D_3$=18279·5 59·9	—	56130·9 (6)	55807·9 (5)	—	48075·2 (2)
—	4d $^4D_4$	$^4D_4$=18219·6	—	—	55867·8 (6)	—	—
4d $^2D_2$	4d $^2D_2$	$^2D_2$=16701·7 100·4	68300·0 (5)	57707·7 (6)	57385·8 (5)	50114·0 (7)	49653·5 (6)
4d $^2D_3$	$^2D_3$	$^2D_3$=16601·3	68397·6 (4)	57808·5 (5)	57486·8 (6)	—	49754·6 (8)
—	—	4d $^4P_1$=13791·7 −158·3	71208·3 (4)	60618·9 (3)	—	—	—
—	—	$^4P_2$=13950·0 −281·6	71049·0 (4)	60460·0 (4)	60139·2 (3)	52865·6 (1)	—
—	—	$^4P_3$=14231·6	70767·9 (5)	—	59855·2 (1)	—	52124·9 (0)
—	—	$^2P_1$=13076·8 196·2	71922·3 (4)	61332·0 (4)	—	53741·2 (1)	53279·1 (0)
—	γ	$^2P_2$=12880·6	—	61528·6 (4)	61207·8 (5)	—	53474·5 (4)

Other regularities

4p......	$^4S_2$ 85000	$^2D_2$ 74409·7	$^2D_3$ 74087·4	$^2P_1$ 66816·7	$^2P_2$ 66355·5
a 12740·0	72264·8 (1)	—	—	54074·2 (0)	53613·3 (5)
b 11758·7	73241·3 (2)	62650·7 (2)	62330·5 (1)	—	—
c 10956·2	74046·1 (2)	63452·6 (2)	63129·7 (2)	—	—
d 10863·7	74133·4 (2)	—	—	55953·5 (4)	55494·2 (1)
e 10761·3	—	63646·8 (4)	63326·4 (0)	—	55595·4 (4)
f 10687·9	—	63720·7 (0)	63400·7 (0)	—	—
g 10259·9	74737·9 (2)	64149·8 (3)	—	56558·9 (2)	—
h 10196·4	74801·6 (3)	64211·6 (3)	63891·2 (0)	—	56160·5 (4)
i 9916·7	—	—	—	56899·3 (1)	56439·5 (0)
j 9429·1	75573·8 (3)	64978·9 (4)	64657·1 (0)	—	—
k 8926·5	—	—	65159·3 (3)	57890·1 (2)	57430·7 (5)
l 8193·6	—	—	—	58622·5 (0)	58162·5 (5)

It will be noticed that there is perfect agreement between the two schemes under consideration as regards the identification of the fundamental multiplets arising from the transition $4p \leftarrow 5s$ except those involving the term $5s\,^2S$. Obviously an inner quantum number 2 must be assigned to the level which is designated as $5s\,^2S$ by Meggers; hence the more probable identification made by Rao has been retained.

In considering the other higher levels, the principle guiding the designation of each has been that a level is assumed to be a quartet or a doublet according as its combinations with $4p\,^4S$ or with ^2D and ^2P are the more intense. However, on account of the relatively large intensities of the intercombination lines, this procedure may not always lead to the correct assignment. The similarity in appearance and behaviour of the lines forming one multiplet has also been of great help in this assignment. The photographing of the whole region from $\lambda\,2800$ to $\lambda\,500$ at once on a single Schumann plate has considerably facilitated the comparison of the behaviour and intensity of the lines although they are widely separated.

Three levels designated as forming $sp^4\,^4P_{1,2,3}$ by Rao have been denominated $4d\,^4F_3$, $^4F_2$, $^4D_1$ by Meggers. Examination of the plates showed that the three lines $\nu\,63976, 63502, 62749$ have an identical appearance and must belong to the same group; particularly on a neon plate taken with a short exposure these lines stand out very prominently. Reference may be made to (d) on the plate. It might be supposed that they form the group $4p\,^4S - 4d\,^4P$ instead of $4p\,^4S - sp^4\,^4P$. The large magnitude of the term supports the latter assignment. Such groups have been discovered to be markedly present in the analogous spectra of N I, O II, F III, S II, Cl III*. In all these the $sp^4\,^4P$ term is inverted. The following table of the magnitudes of the intervals of this term shows clearly that the present assignment is in keeping with the other spectra.

Table 3. Deep quartet term intervals in five-electron systems

	N I	O II	F III	P I	S II	Cl III	As I
$sp^4\,^4P_1$ $^4P_2$ $^4P_3$	$-19{\cdot}6$ $-43{\cdot}9$	-82 -163	-177 -342	— —	$-210{\cdot}1$ $-363{\cdot}1$	-345 -610	$-477{\cdot}3$ $-752{\cdot}1$
$ms\,^4P_1$ $^4P_2$ $^4P_3$	$33{\cdot}8$ $46{\cdot}7$	$105{\cdot}3$ $158{\cdot}6$	$211{\cdot}3$ $318{\cdot}9$	151 249	$270{\cdot}83$ $437{\cdot}00$	$357{\cdot}8$ $520{\cdot}1$	$915{\cdot}8$ $1287{\cdot}7$

This alteration in Meggers's scheme necessitated a complete reassignment of the characteristic terms. Rao has suggested only some of the quartet terms of the $4d$ state. Of his $4d\,^4D$ group, the component $^4D_1$ (marked as doubtful) has been based on the arbitrary choice of a single line. The selection made by the writer seems to be more plausible. Rao and Meggers agree in the identification of the $4d\,^2D$ term, which is well supported by the strong combination lines it gives with $4p\,^2P$ and $4p\,^2D$ and therefore has been adopted by the author.

* I. S. Bowen, *Phys. Rev.* **29**, 237 (1927); **31**, 34 (1928). S. B. Ingram, *Phys. Rev.* **32**, 172 (1928).

The intervals exhibited by the triad of terms $^{4,2}F$, $^{4,2}D$, $^{4,2}P$, arising from the $4d$ state in all spectra of the type under consideration, appear to be very anomalous. Excepting in S II, in which the term is normal, the ^4D term is partially inverted in all these spectra; while the ^4P term shows complete inversion in all except in N I. The scheme now suggested for As I exhibits similar anomalies; the ^4P term is completely inverted and ^4D and ^4F show only partial inversion. In spite of these irregularities in the term intervals, it is curious that the usual order of increasing term values, i.e. ^4P, ^4D, ^4F, is preserved in all cases. This feature is found to hold good in As I also, and perhaps supports the correctness of the proposed classification.

With the uncertainty that is likely to exist in the identification of even the $4d$ terms, on account of the irregularities in the term intervals, it appears nearly impossible, at the present stage, to locate the higher members of this set of terms. There are several characteristic energy levels in the region in which the $5d$ terms may be expected, but these are designated only by arbitrary symbols a, b, etc. It is likely that some at least of these terms belong to the group arising from the addition of a $4d$ electron to the ^1D state of the core.

For the detection of the higher members of the ms series of terms, however, the following table of limits has afforded a valuable clue[*].

Table 4

	N I	O II	F III	P I	S II	Cl III	As I
$ms\ ^4P_1 - {}^4P_3$	80·5	263·8	530·2	400	707·8	877·9	2203·5
$(m+1)s\,^4P_1 - {}^4P_3$	118·7	266·6	—	—	737·8	1185·7	2304·3
...	...	...	...	...	...	...	...
	N II	O III	F IV	P II	S III	Cl IV	As II
Limit $mp\ ^3P_0 - {}^3P_2$	133·9	309	637	469·9	835	1341	2542

Obviously the difference $ms\ ^4P_1 - {}^4P_3$ in each of these spectra tends towards the limiting value $mp\ ^3P_0 - {}^3P_2$ of the next higher ion. Assuming a corresponding feature in the case of As I the location of the group $4p^4S - 6s\,^4P$ giving the total interval of 2304·3 cm.[-1] seems to be very satisfactory. The view that the levels are presumably quartets receives support from the faintness of their combinations with $4p\,^2P$ and $4p\,^2D$. The line $\lambda 1472$ might be mistaken for the second-order neon line $\lambda 736$, but on a He plate on which Ne lines are very faint the two are distinctly resolved. Attempts to locate the third member of the series were not successful but it seemed clear that it lies beyond $\nu 76,000$. In this region several lines of As II also occur so that the isolation of the lines belonging to As I has, in itself, been difficult.

For completeness the following supermultiplet in the infra-red, due to the transition $5s \leftarrow 5p$ of As I, is shown also. The identification of these 25 lines is due entirely to Meggers and de Bruin.

* I. S. Bowen, *loc. cit.* S. B. Ingram, *loc. cit.* A. S. Rao (As II), *Proc. Phys. Soc.* **44**, 343 (1932).

Table 5. As I, supermultiplet $5p \rightarrow 5s$

5p \ 5s...	$^4P_1$ 34308·0	$^4P_2$ 33392·2	$^4P_3$ 32104·5	$^2P_1$ 31866·8	$^2P_2$ 30397·2
	915·8	*1287·7*	*237·7*	*1469·6*	
$^4P_1$ 24141·7	10166·27 (80)	—	—	—	—
723·5					
$^4P_2$ 23418·2	—	9973·35 (100)	—	—	—
1389·2					
$^4P_3$ 22029·0	—	—	10074·81 (150)	—	—
$^4D_1$ 22975·5	11332·50 (150)	10416·05 (10)	—	—	—
371·6					
$^4D_2$ 22603·9	11704·13 (50)	10787·68 (25)	—	—	—
885·0					
$^4D_3$ 21718·9	—	11672·62 (100)	10384·94 (8)	—	—
1167·0					
$^4D_4$ 20551·9	—	—	11551·96 (150)	—	—
$^4S_2$ 21354·8	—	12036·73 (50)	10749·03 (50)	—	—
$^2D_2$ 22447·4	11860·63 (100)	10944·13 (15)	—	9418·9 (25)	—
1614·8					
$^2D_3$ 20832·6	—	12558·95 (25)	11271·26 (100)	—	95640·0 (60)
$^2P_1$ 20749·7	—	—	—	11116·62 (20)	—
71·7					
$^2P_2$ 20678·0	—	12713·57 (4)	—	11188·15 (50)	—
$^2S_1$ 19900·5	—	13491·27 (8)	—	11965·60 (10)	—

Meggers and de Bruin have called attention to a considerable number of fairly strong lines of As I, which they have been able to detect in the blue and green regions of the spectrum; two of these, $\lambda 5361·12$ (10) and $\lambda 5497·10$ (4), have been interpreted as the forbidden combinations $4p\,^4S_2 - 4p\,^2P_{1,2}$ and the suggestion has been made that these lines might be of the same character as the nebular lines. Two other lines in the extreme ultra-violet, $\lambda 1612$ and $\lambda 1558$, have also been suggested as being probably the forbidden combinations $4p\,^4S_2 - 5p\,^4D_{1,3}$. Neither the writer nor K. R. Rao has been able to photograph any of the lines in the visible region. The lines $\lambda 1612$ and $\lambda 1558$ have been otherwise classified in the present scheme and it is believed that the relations mentioned by Meggers are only accidental.

§4. TERM VALUES

The question of the determination of the absolute term values and the ionization potential of As I has an additional importance on account of the recent interesting observation made by K. R. Rao and J. S. Badami* of the occurrence of the Lyman series of hydrogen in a hollow cathode discharge in He, as a resonance spectrum, in the presence of arsenic. The earliest estimate of this ionization potential was made by Ruark and others† from experiments on low-voltage arcs in arsenic vapour: they

* K. R. Rao and J. S. Badami, *Nature*, October 3, 1931.
† A. E. Ruark and others, *Bur. Standards Sci. Papers*, **19**, 463 (1924).

arrived at the value $11 \cdot 54 = 0 \cdot 5$ V. Rao has provisionally adopted this experimental value and based all the terms on $4p\,^4S_2 = 93,500$ cm.$^{-1}$ Meggers and de Bruin made an interesting comparison of the centres of gravity of the $5s$ terms in the arc spectra of Ga, Ge, As, etc. and, by interpolation, assumed an approximate value of 30,000 cm.$^{-1}$ for $5s\,^4P_1$ which led to $4p\,^4S_2 = 80,697$. In the present work two members of several series are available for the calculation of limits. The series $4p\,^4S - ms\,^4P$ gives a limit equal to $81,980$, but if the third member lies, as suggested in a preceding paragraph, beyond $\nu\,76,000$, the limit $4p\,^4S_2$ probably will be larger than $84,000$. The series $5s\,^4P - mp\,^4S$ gives $4p\,^4S = 88,150$. If the identification of the doublets due to transition $4p \leftarrow 6s$ be correct, they lead to the value $4p\,^4S_2 = 81,820$ cm.$^{-1}$ Considering these various estimates, a mean value $4p\,^4S_2 = 85,000$ cm.$^{-1}$ appears to be fairly probable; the error may not perhaps exceed about 2000 units. This mean value is also in keeping with the following variation of the ionization potential with atomic number; the number below each element is the value of the first ionization potential of the element*.

Table 6

B	C	N	O	F
8·34	11·2	14·48	13·56	18·6
Al	Si	P	S	Cl
5·95	8·12	—	10·31	12·96
Ga	Ge	As	Se	Br
5·97	8·09	10·5	9·70	11·80

In table 7 the term values in As I are calculated on the assumption that $4p\,^4S_2 = 85,000$ cm.$^{-1}$ The ionization potential is approximately equal to $10 \cdot 5$ V.

Finally, in table 8 is given a catalogue of all the lines which are ascribed to the neutral atom of arsenic, together with the classification of each, as it is proposed in the present investigation. The first 25 lines in the infra-red are due to Meggers and de Bruin and the intensities of these are on the scale of 1000 for the maximum. Of the remaining lines, the wave-lengths down to $\lambda 1563$, as measured by K. R. Rao, have been adopted as they were obtained from larger dispersion plates. There are, however, several additional lines in this region, not previously recorded. These and all the lines below $\lambda 1563$ were measured by the writer. For the sake of uniformity in comparison, the intensities of all the lines (except those in the infra-red) are visual estimates made carefully by the writer from the new plates. Lines which are probably to be ascribed to As I are present on the plates even below $\lambda 1319$, extending as

* B I: I. S. Bowen, *Phys. Rev.* **29**, 231 (1927). N I: S. B. Ingram, *Phys. Rev.* **34**, 421 (1929). S I: J. J. Hopfield and G. H. Dieke, *Phys. Rev.* **27**, 638 (1926). Se I: R. C. Gibbs and Ruedy, *Phys. Rev.* **37**, 1704 (1931). Al and Ga: A. Fowler, *Report on Line Spectra*. C I and Si I: A. Fowler and E. W. H. Selwyn; and A. Fowler, *Proc. R. S.* **118**, 34 (1928); **123**, 422 (1929). Ge I: K. R. Rao, *Proc. R. S.* **124**, 465 (1929). F I: H. Dingle, *Proc. R. S.* **117**, 407 (1928). Cl I and Br I: C. C. Kiess and T. L. de Bruin, *Bur. Standards J. Res.* **2**, 1117 (1929); **4**, 668 (1930). O I: J. J. Hopfield, *Nature*, **112**, 437 (1923).

far down as $\lambda 1250$. But as they could not confidently be distinguished from those of As II occurring in that region, they have been omitted from the table.

Table 7. Term values of As I

Term	Term value	Term	Term value
$4p\ ^4S_2$	85000	$4d\ ^4D_2$	18519·0
$^2D_2$	74409·7	$4d\ ^4D_1$	18416·7
$^2D_3$	74087·4	$4d\ ^4D_3$	18279·5
$^2P_1$	66816·7	$4d\ ^4D_4$	18219·6
$^2P_2$	66355·5	$6s\ ^4P_1$	17995·4
$5s\ ^4P_1$	34308·0	$4d\ ^2F_4$	17224·6
$^4P_2$	33392·2	$6s\ ^4P_2$	17083·2
$^4P_3$	32104·5	$4d\ ^2D_2$	16701·7
$^2P_1$	31866·8	$6s\ ^2P_1$	16689·0
$^2P_2$	30397·2	$4d\ ^2D_3$	16601·3
$^2D_3$	24186·9	$6s\ ^4P_3$	15691·1
$^2D_2$	24167·8	$6s\ ^2P_2$	15306·5
$5p\ ^4P_1$	24141·7	$4d\ ^4P_3$	14231·6
$^4P_2$	23418·2	$4d\ ^4P_2$	13950·0
$^4D_1$	22975·5	$4d\ ^4P_1$	13791·7
$^4D_2$	22603·9	$4d\ ^2P_1$	13076·8
$^2D_2$	22447·4	$4d\ ^2P_2$	12880·6
$sp^4\ ^4P_3$	22251·9	a	12740·0
$5p\ ^4P_3$	22029·0	$sp^4\ ^2D_2$	12486·7
$5p\ ^4D_3$	21718·9	$sp^4\ ^2D_3$	12397·7
$sp^4\ ^4P_2$	21499·8	$sp^4\ ^2P_2$	12224·2
$5p\ ^4S_2$	21354·8	b	11758·7
$sp^4\ ^4P_1$	21022·5	$sp^4\ ^2S_1$	11371·9
$5p\ ^2D_3$	20832·6	c	10956·2
$5p\ ^2P_1$	20749·7	d	10863·7
$5p\ ^2P_2$	20678·0	e	10761·3
$4d\ ^4F_3$	20661·1	f	10687·9
$5p\ ^4D_4$	20551·9	g	10259·9
$4d\ ^4F_2$	20192·9	h	10196·4
$4d\ ^4F_4$	20042·7	i	9916·7
$5p\ ^2S_1$	19900·5	j	9429·1
$5s\ ^2S_1$	19505·2	k	8926·5
$4d\ ^2F_3$	19045·2	l	8193·6

§5. SUMMARY OF CONCLUSIONS

About 240 lines have been ascribed to the neutral atom of arsenic, of which nearly 100 lines have been newly recorded, the majority of these having been classified. The spectrum was excited by the method of the hollow cathode discharge in He or Ne. The doublet and quartet systems, characteristic of the spectrum, which have been previously published by K. R. Rao and by Meggers and de Bruin, have been discussed in detail and considerably altered and extended in the light of the more complete data obtained by the above method in the Schumann region.

Several new levels have been added; the ^4P term involving a transition of one of the inner electrons has been confirmed. Higher members of the chief groups of the ms series of terms have been identified. Interesting anomalies have been detected in the intervals of the $4d$ terms, although the usual order of increasing term

Table 8. Catalogue of lines of As I

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
10614		9418·9	$5s\ ^2P_1-5p\ ^2D_2$	—	—
10453		9564·0	$5s\ ^2P_2-5p\ ^2D_3$	—	—
10023·98	(4)	9973·35	—	—	$5s\ ^4P_2-5p\ ^4P_2$
9923·03	(5)	10074·81	—	—	$5s\ ^4P_3-5p\ ^4P_3$
9833·76	(5)	10166·27	—	—	$5s\ ^4P_1-5p\ ^4P_1$
9826·69	(8)	10384·94	—	—	$5s\ ^4P_3-5p\ ^4D_3$
9597·94	(10)	10416·05	—	—	$5s\ ^4P_2-5p\ ^4D_1$
9300·62	(50)	10749·03	—	—	$5s\ ^4P_3-5p\ ^4S_2$
9267·29	(25)	10787·68	—	—	$5s\ ^4P_2-5p\ ^4D_2$
9134·81	(15)	10944·13	—	$5s\ ^4P_2-5p\ ^2D_2$	—
8993·08	(20)	11116·62	$5s\ ^2P_1-5p\ ^2P_1$	—	—
8935·58	(50)	11188·15	$5s\ ^2P_1-5p\ ^2P_2$	—	—
8869·69	(100)	11271·26	—	$5s\ ^4P_3-5p\ ^2D_3$	—
8821·76	(150)	11332·50	—	—	$5s\ ^4P_1-5p\ ^4D_1$
8654·16	(100)	11551·96	—	—	$5s\ ^4P_3-5p\ ^4D_4$
8564·71	(100)	11672·62	—	—	$5s\ ^4P_2-5p\ ^4D_3$
8541·65	(50)	11704·13	—	—	$5s\ ^4P_1-5p\ ^4D_2$
8428·94	(100)	11860·63	—	$5s\ ^4P_1-5p\ ^2D_2$	—
8355·00	(10)	11965·60	$5s\ ^2P_1-5p\ ^2S_1$	—	—
8305·62	(50)	12036·73	—	—	$5s\ ^4P_1-5p\ ^4S_2$
8055·72	(5)	12410·13	—	—	$5s\ ^2D_2-X$
8042·95	(10)	12429·83	—	—	$5s\ ^2D_3-X$
7960·26	(25)	12558·95	—	$5s\ ^4P_2-5p\ ^2D_3$	—
7863·45	(4)	12713·57	—	$5s\ ^4P_2-5p\ ^2P_2$	—
7410·07	(8)	13491·43	—	$5s\ ^4P_2-5p\ ^2S_1$	—
3119·576	(4)	32046·38	—	$4p\ ^2P_2-5s\ ^4P_1$	—
3075·317	(2)	32507·56	—	$4p\ ^2P_1-5s\ ^4P_1$	--
3032·845	(4)	32962·77	—	$4p\ ^2P_2-5s\ ^4P_2$	—
2990·991	(2)	33423·91	—	$4p\ ^2P_1-5s\ ^4P_2$	—
2918·850	(0)	34250·3	—	$4p\ ^2P_2-5s\ ^4P_3$	—
2898·702	(5)	34488·12	$4p\ ^2P_2-5s\ ^2P_1$	—	—
2860·452	(7)	34949·27	$4p\ ^2P_1-5s\ ^2P_1$	—	—
2780·197	(8)	35958·09	$4p\ ^2p_2-5s\ ^2P_2$	—	—
2744·991	(6)	36419·24	$4p\ ^2P_1-5s\ ^2P_2$	—	—
2492·89	(5)	40102·0	—	$4p\ ^2D_2-5s\ ^4P_1$	—
2456·53	(5)	40695·6	—	$4p\ ^2D_3-5s\ ^4P_2$	—
2437·22	(4)	41018·0	—	$4p\ ^2D_2-5s\ ^4P_2$	—
2381·19	(6)	41983·0	—	$4p\ ^2D_3-5s\ ^4P_3$	—
2370·80	(7)	42167·0	$4p\ ^2P_2-5s\ ^2D_3$	—	—
2369·67	(7)	42187·1	$4p\ ^2P_2-5s\ ^2D_2$	—	—
2363·04	(3)	42305·4	—	$4p\ ^2D_2-5s\ ^4P_3$	—
2349·83	(9)	42543·2	$4p\ ^2D_2-5s\ ^2P_1$	—	—
2344·02	(5)	42648·6	$4p\ ^2P_1-5s\ ^2D_2$	—	—
2288·12	(10)	43690·5	$4p\ ^2D_3-5s\ ^2P_2$	—	—
2271·39	(3)	44012·4	$4p\ ^2D_2-5s\ ^2P_2$	—	—
2266·75	(2)	44102·4	—	$4p\ ^2P_2-sp^4\ ^4P_3$	—
2228·72	(3)	44854·8	—	$4p\ ^2P_2-sp^4\ ^4P_2$	—
2206·04	(2)	45315·9	—	$4p\ ^2P_1-sp^4\ ^4P_2$	—
2205·20	(2)	45333·2	—	$4p\ ^2P_2-sp^4\ ^4P_1$	—
2187·83	(1)	45694·7	—	$4p\ ^2P_2-4d\ ^4F_3$	—
2182·98	(2)	45794·6	—	$4p\ ^2P_1-sp^4\ ^4P_1$	—
2176·26	(1)	45935·9	—	—	—
2165·57	(6)	46162·6	—	$4p\ ^2P_2-4d\ ^4F_2$	—
2144·14	(5)	46624·0	—	$4p\ ^2P_1-4d\ ^4F_2$	—
2138·53	(2)	46746·3	—	—	—
2133·77	(4)	46850·6	$4p\ ^2P_2-5s\ ^2S_1$	—	—
2124·06	(1)	47064·7	—	—	—
2112·99	(5)	47311·3	$4p\ ^2P_1-5s\ ^2S_1$	—	—
2089·68	(2)	47838·9	—	$4p\ ^2P_2-4d\ ^4D_2$	—

Table 8. Catalogue of lines of As I (*continued*)

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
2085·33	(3)	47938·7	—	$4p\ ^2P_2-4d\ ^4D_1$	—
2079·41	(2)	48075·2	—	$4p\ ^2P_2-4d\ ^4D_3$	—
2069·79	(3)	48298·6	—	$4p\ ^2P_1-4d\ ^4D_2$	—
2068·34	(3)	48332·4	—	—	—
2067·13	(1)	48360·7	—	$4p\ ^2P_2-6s\ ^4P_1$	—
2065·42	(3)	48400·8	—	$4p\ ^2P_1-4d\ ^4D_1$	—
2047·58	(2)	48822·4	—	$4p\ ^2P_1-6s\ ^4P_1$	—
2028·96	(0)	49272·6	—	$4p\ ^2P_2-6s\ ^4P_2$	—
2025·36	(1)	49358·0	—	—	—
2013·30	(6)	49653·5	$4p\ ^2P_2-4d\ ^2D_2$	—	—
2012·74	(4)	49667·4	$4p\ ^2P_2-6s\ ^2P_1$	—	—
2010·05	(8)	49733·8	—	$4p\ ^2P_1-6s\ ^4P_2$	—
2009·21	(8)	49754·6	$4p\ ^2P_2-4d\ ^2D_3$	—	—
2003·28	(10)	49901·4	$4p\ ^2D_3-5s\ ^2D_3$	—	—
2002·53	(4)	49920·6	$4p\ ^2D_3-5s\ ^2D_2$	—	—
λ (vac.)					
1995·45	(7)	50114·0	$4p\ ^2P_1-4d\ ^2D_2$	—	—
1994·87	(5)	50128·6	$4p\ ^2P_1-6s\ ^2P_1$	—	—
1991·08	(7)	50224·0	$4p\ ^2D_2-5s\ ^2D_3$	—	—
1990·37	(9)	50241·9	$4p\ ^2D_2-5s\ ^2D_2$	—	—
1973·80	(0)	50663·7	—	$4p\ ^2P_2-6s\ ^4P_3$	—
1972·64	(10)	50693·5	—	—	$4p\ ^4S_2-5s\ ^4P_1$
1958·91	(5)	51048·8	$4p\ ^2P_2-6s\ ^2P_2$	—	—
1941·52	(2)	51506·0	$4p\ ^2P_1-6s\ ^2P_2$	—	—
1937·68	(12)	51608·1	—	—	$4p\ ^4S_2-5s\ ^4P_2$
1929·17	(1)	51835·7	—	$4p\ ^2D_3-sp^4\ ^4P_3$	—
1918·47	(0)	52124·9	—	$4p\ ^2P_2-4d\ ^4P_3$	—
1917·24	(3)	52158·3	—	$4p\ ^2D_2-sp^4\ ^4P_3$	—
1902·48	(2)	52563·0	—	$4p\ ^2P_2-4d\ ^4P_1$	—
1901·59	(2)	52587·6	—	$4p\ ^2D_3-sp^4\ ^4P_2$	—
1900·02	(0)	52631·0	—	—	—
1891·59	(1)	52865·6	—	$4p\ ^2P_1-4d\ ^4P_2$	—
1890·53	(15)	52895·2	—	—	$4p\ ^4S_2-5s\ ^4P_3$
1882·03	(4)	53134·1	—	$4p\ ^4S_2-5s\ ^2P_1$	—
1876·91	(0)	53279·1	$4p\ ^2P_2-4d\ ^2P_1$	—	—
1873·09	(4)	53387·7	—	$4p\ ^2D_2-sp^4\ ^4P_1$	—
1871·74	(4)	53426·2	—	$4p\ ^2D_3-4d\ ^4F_3$	—
1870·05	(4)	53474·5	$4p\ ^2P_2-4d\ ^2P_2$	—	—
1865·21	(5)	53613·3	—	—	$4p\ ^2P_2-a$
1860·77	(1)	53741·2	$4p\ ^2P_1-4d\ ^2P_1$	—	—
1860·48	(6)	53749·6	—	$4p\ ^2D_2-4d\ ^4F_3$	—
1855·48	(3)	53894·4	—	$4p\ ^2D_3-4d\ ^4F_2$	—
1853·29	(4)	53958·1	$4p\ ^2P_2-sp^4\ ^2D_3$	—	—
1850·32	(6)	54044·7	—	$4p\ ^2D_3-4d\ ^4F_4$	—
1849·31	(0)	54074·2	—	—	$4p\ ^2P_1-a$
1847·39	(5)	54130·4	$4p\ ^2P_2-sp^4\ ^2P_2$	—	—
1844·49	(5)	54215·5	—	$4p\ ^2D_2-4d\ ^4F_2$	—
1840·58	(4)	54330·7	$4p\ ^2P_1-sp^4\ ^2D_2$	—	—
1831·80	(5)	54591·1	$4p\ ^2P_1-sp^4\ ^2P_2$	—	—
1831·39	(6)	54603·3	—	$4p\ ^4S_2-5s\ ^2P_2$	—
1821·44	(4)	54901·6	$4p\ ^2D_2-5s\ ^2S_1$	—	—
1818·72	(2)	54983·7	$4p\ ^2P_2-sp^4\ ^2S_1$	—	—
1816·79	(4)	55042·1	$4p\ ^2D_3-4d\ ^2F_3$	—	—
1813·18	(1)	55151·8	—	—	—
1810·73	(1)	55042·1	—	—	—
1806·21	(9)	55364·5	$4p\ ^2D_2-4d\ ^2F_3$	—	—
1803·60	(2)	55444·7	$4p\ ^2P_1-sp^4\ ^2S_1$	—	—
1801·99	(1)	55494·2	—	—	$4p\ ^2P_2-d$
1799·61	(3)	55567·6	—	$4p\ ^2D_3-4d\ ^4D_2$	—

Table 8. Catalogue of lines of As I (*continued*)

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other com-binations
1798·71	(4)	55595·4	—		$4p\ ^2P_2-e$
1791·86	(5)	55807·9	—	—	
1789·94	(6)	55867·8	—	$4p\ ^2D_3-4d\ ^4D_3$	—
1789·25	(7)	55889·4	—	$4p\ ^2D_3-4d\ ^4D_4$	—
1787·20	(4)	55953·5	—	$4p\ ^2D_2-4d\ ^4D_2$	
1785·97	(3)	55992·0	—		$4p\ ^2P_1-d$
1781·55	(6)	56130·9	—	$4p\ ^2D_2-4d\ ^4D_1$	—
1780·61	(4)	56160·5	—	$4p\ ^2D_2-4d\ ^4D_3$	—
1775·11	(1)	56334·5	—		$4p\ ^2P_2-h$
1772·63	(2)	56413·3	—	—	—
1771·81	(0)	56439·5	—	$4p\ ^2D_2-6s\ ^4P_1$	—
1768·07	(2)	56558·9	—	—	$4p\ ^2P_2-i$
1760·39	(1)	56805·6	—	—	$4p\ ^2P_1-g$
1758·62	(9)	56862·8	$4p\ ^2D_3-4d\ ^2F_4$	—	—
1757·49	(1)	56899·3	—	—	—
1756·37	(00)	56935·6	—	—	$4p\ ^2P_1-i$
1754·24	(3)	57004·7	—	$4p\ ^2D_3-6s\ ^4P_2$	—
1746·15	(1)	57268·8	—	—	—
1742·59	(5)	57385·8	$4p\ ^2D_3-4d\ ^2D_2$	—	—
1741·23	(5)	57430·7	—	—	$4p\ ^2P_2-k$
1739·53	(6)	57486·8	$4p\ ^2D_3-4d\ ^2D_3$	—	—
1735·52	(1)	57619·6	—	—	—
1732·87	(6)	57707·7	$4p\ ^2D_2-4d\ ^2D_2$	—	—
1732·51	(6)	57719·7	$4p\ ^2D_2-6s\ ^2P_1$	—	—
1729·85	(5)	57808·5	$4p\ ^2D_2-4d\ ^2D_3$	—	—
1727·41	(2)	57890·1	—	—	$4p\ ^2P_1-k$
1724·84	(3)	57976·4	—	—	—
1719·32	(5)	58162·5	—	—	$4p\ ^2P_2-l$
1712·41	(2)	58397·2	—	$4p\ ^2D_3-6s\ ^4P_3$	—
1710·25	(0)	58471·0	—	—	—
1709·42	(1)	58499·4	—	—	—
1705·83	(0)	58622·5	—	—	$4p\ ^2P_1-l$
1703·05	(1)	58718·2	—	$4p\ ^2D_2-6s\ ^4P_3$	—
1701·22	(8)	58781·4	$4p\ ^2D_3-6s\ ^2P_2$	—	—
1697·35	(4)	58915·4	—	—	—
1691·89	(5)	59105·5	$4p\ ^2D_2-6s\ ^2P_2$	—	—
1680·50	(2)	59506·1	—	—	—
1670·70	(1)	59855·2	—	$4p\ ^2D_3-4d\ ^4P_3$	—
1662·81	(3)	60139·2	—	$4p\ ^2D_3-4d\ ^4P_2$	—
1653·99	(4)	60460·0	—	$4p\ ^2D_2-4d\ ^4P_2$	—
1649·65	(3)	60618·9	—	$4p\ ^2D_2-4d\ ^4P_1$	—
1644·40	(5)	60812·5	—	$4p\ ^4S_2-5s\ ^2D_3$	—
1643·84	(5)	60833·2	—	$4p\ ^4S_2-5s\ ^2D_2$	—
1633·78	(5)	61207·8	$4p\ ^2D_3-4d\ ^2P_2$	—	—
1630·47	(4)	61332·0	$4p\ ^2D_2-4d\ ^2P_1$	—	—
1625·26	(4)	61528·6	$4p\ ^2D_2-4d\ ^2P_2$	—	—
1623·38	(4)	61599·9	$4p\ ^2D_3-sp^4\ ^2D_2$	—	—
1621·02	(6)	61689·6	$4p\ ^2D_3-sp^4\ ^2D_3$	—	—
1618·06	(1)	61802·4	—	—	—
1616·56	(5)	61859·8	$4p\ ^2D_3-sp^4\ ^2P_2$	—	—
1614·92	(5)	61922·6	$4p\ ^2D_2-sp^4\ ^2D_2$	—	—
1612·59	(7)	62012·0	$4p\ ^2D_2-sp^4\ ^2D_3$	—	—
1608·17	(6)	62182·4	$4p\ ^2D_2-sp^4\ ^2P_2$	—	—
1604·35	(1)	62330·5	—	—	$4p\ ^2D_3-b$
1598·59	(2)	62555·1	—	—	—
1597·58	(2)	62594·7	—	—	—
1596·15	(2)	62650·7	—	—	$4p\ ^2D_2-b$
1593·66	(9)	62748·6	—	—	$4p\ ^4S_2-sp^4\ ^4P_3$
1587·36	(5)	62997·7	—	—	—

Table 8. Catalogue of lines of As I (*continued*)

λ		ν	Classification		
			Doublets	Inter-combinations	Quartets and other combinations
1584·04	(2)	63129·7	—	—	$4p\ ^2D_3 - c$
1579·12	(0)	63326·4	—	—	$4p\ ^2D_3 - e$
1577·27	(0)	63400·7	—	—	$4p\ ^2D_3 - f$
1575·98	(2)	63452·6	—	—	$4p\ ^2D_2 - c$
1574·75	(8)	63502·2	—	—	$4p\ ^4S_2 - sp^4\ ^4P_2$
1573·97	(6)	63533·6	—	—	
1571·17	(4)	63646·8	—	—	$4p\ ^2D_2 - e$
1569·35	(0)	63720·7	—	—	$4p\ ^2D_2 - f$
1568·29	(1)	63763·7	—	—	—
1566·51	(2)	63836·2	—	—	—
1565·16	(0)	63891·2	—	—	$4p\ ^2D_3 - h$
1563·08	(7)	63976·3	—	—	$4p\ ^4S_2 - sp^4\ ^4P_1$
1559·53	(6)	64121·9	—	—	—
1558·85	(3)	64149·8	—	—	$4p\ ^2D_2 - g$
1557·35	(3)	64211·6	—	—	$4p\ ^2D_2 - h$
1556·25	(3)	64257·0	—	—	—
1554·30	(4)	64337·6	—	—	$4p\ ^4S_2 - 4d\ ^4F_3$
1547·60	(2)	64616·2	—	—	—
1546·62	(0)	64657·1	—	—	$4p\ ^2D_3 - j$
1543·01	(4)	64808·4	—	—	$4p\ ^4S_2 - 4d\ ^4F_2$
1538·94	(4)	64978·9	—	—	$4p\ ^2D_2 - j$
1534·70	(3)	65159·3	—	—	$4p\ ^2D_3 - k$
1533·74	(2)	65200·1	—	—	—
1528·95	(0)	65404·4	—	—	—
1526·90	(3)	65492·2	—	$4p\ ^4S_2 - 5s\ ^2S_1$	—
1515·58	(4)	65981·3	—	—	—
1510·71	(3)	66194·0	—	—	—
1509·77	(1)	66235·3	—	—	—
1504·22	(5)	66479·7	—	—	$4p\ ^4S_2 - 4d\ ^4D_2$
1501·87	(3)	66583·7	—	—	$4p\ ^4S_2 - 4d\ ^4D_1$
1492·45	(6)	67003·9	—	—	$4p\ ^4S_2 - 6s\ ^4P_1$
1472·41	(7)	67915·9	—	—	$4p\ ^4S_2 - 6s\ ^4P_2$
1464·13	(5)	68300·0	—	$4p\ ^4S_2 - 4d\ ^2D_2$	—
1463·91	(5)	68310·1	—	$4p\ ^4S_2 - 6s\ ^2P_1$	—
1462·04	(4)	68397·6	—	$4p\ ^4S_2 - 4d\ ^2D_3$	—
1442·81	(8)	69309·2	—	—	$4p\ ^4S_2 - 6s\ ^4P_3$
1434·82	(8)	69695·1	—	$4p\ ^4S_2 - 6s\ ^2P_2$	—
1423·03	(6)	70272·6	—	—	—
1413·07	(5)	70767·9	—	—	$4p\ ^4S_2 - 4d\ ^4P_3$
1407·48	(4)	71049·0	—	—	$4p\ ^4S_2 - 4d\ ^4P_2$
1404·33	(4)	71208·3	—	—	$4p\ ^4S_2 - 4d\ ^4P_1$
1402·67	(0)	71292·6	—	—	—
1390·39	(4)	71922·3	—	$4p\ ^4S_2 - 4d\ ^2P_1$	—
1389·72	(0)	71956·9	—	—	—
1383·80	(1)	72264·8	—	—	$4p\ ^4S_2 - a$
1379·05	(4)	72513·7	—	$4p\ ^4S_2 - sp^4\ ^2D_2$	—
1377·37	(3)	72602·1	—	$4p\ ^4S_2 - sp^4\ ^2D_3$	—
1375·91	(1)	72679·2	—	—	—
1373·92	(4)	72784·4	—	$4p\ ^4S_2 - sp^4\ ^2P_2$	—
1365·35	(2)	73241·3	—	—	$4p\ ^4S_2 - b$
1363·35	(0)	73348·7	—	—	—
1350·51	(2)	74046·1	—	—	$4p\ ^4S_2 - c$
1348·92	(2)	74133·4	—	—	$4p\ ^4S_2 - d$
1338·36	(2)	74718·3	—	—	—
1338·01	(2)	74737·9	—	—	$4p\ ^4S_2 - g$
1336·87	(3)	74801·6	—	—	$4p\ ^4S_2 - h$
1325·36	(2)	75451·2	—	—	—
1323·21	(3)	75573·8	—	—	$4p\ ^4S_2 - j$
1319·48	(2)	75787·4	—	—	—

values, i.e. ^4P, ^4D, ^4F, has been found to exist. A mean value of 85,000 cm.$^{-1}$ has been suggested for the deepest term $4p\ ^4S_2$ which leads to a first ionization potential of approximately 10·5 V. for arsenic.

§6. ACKNOWLEDGMENTS

OBITUARY NOTICES

D. W. DYE, D.Sc., F.R.S.

DAVID WILLIAM DYE, whose death occurred on February 18, 1932, was an acknowledged authority on the subject of precision electrical measurements, and it may be said with but little exaggeration that he devoted his whole life to work in this field.

The third son of Charles Dye, J.P., of Portsmouth, he was born on December 30, 1887, and received his education at the Portsmouth Municipal Technical College, and later at the City and Guilds Engineering College, South Kensington, where he graduated at London University. He then took a short apprenticeship course with the British Thomson-Houston Company at Rugby, and in 1910 joined the staff of the National Physical Laboratory, where he worked until his death.

Dye's earliest work at the National Physical Laboratory was done under the direction of Albert Campbell, who was then in charge of the Electrical Measurements Division, and he was associated with Campbell in the development of methods for the magnetic testing of iron and its alloys in various forms, the construction of standards of inductance, and the measurement of currents of radio-frequency. When Campbell retired, Dye took charge of this side of the work of the Laboratory, and soon after the retirement of Mr F. E. (now Sir Frank) Smith in 1919 he became head of the Electrical Standards and Measurements Division. In this position he was responsible for the fundamental standards of current, e.m.f., resistance, inductance, capacity and frequency, as well as for the magnetic work. He soon showed that he had a wonderful instinct for measurements of the very highest accuracy, and especially for the attainment of this accuracy by means of perfection of the mechanical construction of his instruments.

His most notable achievement was undoubtedly the setting up of the fundamental standards of frequency at present in use at the National Physical Laboratory, and its extent may be judged from the fact that when he started this work about ten years ago radio-frequencies could not be measured with an accuracy better than one part in a thousand, but in the year of his death he completed and installed standardizing equipment, which operates continuously, and by means of which it is possible to determine the frequency of the Laboratory standards at any time with an accuracy of one part in ten million. This accuracy is obtained simply by inspection of a chronograph chart on which a record is made automatically.

The course followed by this investigation throws considerable light on his methods of working. He took the Eccles valve-maintained tuning-fork, carefully studied its possibilities and limitations by a long series of observations, and finally converted it into a precision time-standard[1], by means of which an audible frequency could be produced with an accuracy previously unobtainable. He then investigated the Abraham-Bloch multivibrator circuit for the generation of har-

D. W. DYE, D.SC., F.R.S.

monics of a high order from a known fundamental frequency, determined its properties by a long series of careful measurements, and then set himself to control its fundamental frequency by means of his tuning-fork standard. In this way all the harmonics up to the 120th, i.e. frequencies up to 120,000, became known with the same high accuracy that had been obtained for the fundamental frequency of 1000. Then by making the 20th harmonic of the first multivibrator control a second one of higher frequency, he obtained standardized harmonic frequencies up to 1,200,000 ∼. In each case the advance was made by means of perfection of instrumental technique based on a long series of accurate observations covering a wide range of possible conditions of use. The result was a standard wave-meter [2] which now forms our national standard, and has no superior. In recent years he had made many improvements in the original model. Although the standard tuning-fork was made of elinvar it was found to have an appreciable temperature coefficient. Later, as the accuracy of measurement increased, it was found to vary slightly in frequency with atmospheric pressure. It was, therefore, mounted in a massive air-tight enclosure, the temperature of which was accurately controlled. The idea followed of obtaining a continuous record of the frequency in terms of the standard clock. Current from the standard tuning-fork was made to drive a chronograph by means of a small synchronous motor. Signals from the pendulum of the standard clock were made to operate the pencil of this chronograph, and at intervals of one hour a contact was closed and these signals were recorded as dots on the chronograph chart. By means of suitable gearing, the chronograph drum was caused to make one revolution per second, and the marking pencil was traversed 1 mm. per hour in a direction parallel to the axis of the drum. Thus the record consists of lines of dots 1 mm. apart, and if the line is parallel with the axis of the drum, the rate of the tuning-fork is zero with reference to the standard clock. A rate of one part in a million tilts the line so that it makes an angle of about 30° to the axis.

At the time of his death, Dye was working on a new form of standard vibrator which was intended to supplement the tuning-fork. The new standard took the form of a quartz ring, cut so that it could be made to vibrate radially by piezo-electric excitation. He considered that the simple nature of the stresses in such a ring would make it a standard of the greatest possible stability, and the experimental results support this view. His important paper in the *Proceedings of the Physical Society* on "The Piezo-Electric Quartz Resonator and its Equivalent Electrical Circuit" [3] will be recognized as forming the groundwork of accurate measurements on which this standard was based. He had also made a detailed study, as yet unpublished, of the vibrations of piezo-electric quartz crystals, using for this purpose the methods of optical interferometry.

Ranking in importance with his frequency standards is the magnetometer [4] which he devised for the measurement of the vertical component of the earth's magnetic field. Writing of this instrument, Mr Albert Campbell says: "His beautiful magnetometer was a great step in advance of its predecessors, for it showed that the methods which had been trusted as standard for many years back were in very

considerable error. This puzzled the authorities very much, but in the end the sources of error were tracked down, and the Dye instrument will, without doubt, be the unassailable standard for many years to come." The instrument now forms a principal part of the equipment of the Abinger Magnetic Observatory.

His investigations of the properties of inter-valve transformers at telephonic frequencies [5], the use of the Schering bridge at radio-frequencies, the measurement of the effective resistances of standard condensers at radio-frequencies [6], the use of an electrically-driven sonometer as an audio-frequency meter, and the design and construction of a standard variable air condenser, though not of the same fundamental importance as those previously mentioned, are characterized by the same precision of measurement and refinement of instrumental construction. Dye himself had acquired great skill in mechanical construction, and he delighted in doing the more refined mechanical work with his own hands: indeed, he seemed never so happy as when giving the last touches to the grinding of a quartz crystal, or rolling out a piece of the finest Wollaston wire for the purpose of making a delicate suspension, unless perhaps it was when demonstrating the finished instrument.

Dye was awarded the degree of Doctor of Science of London University in 1926, and was elected a Fellow of the Royal Society in 1928 and of the City and Guilds Institute in 1931. He was a member of the Council of the Physical Society, of the Radio Research Board and of the Comité Consultatif d'Électricité of the Bureau Internationale des Poids et Mesures, and also secretary of the British National Committee of the Union Radio-Scientifique Internationale. He served as chairman of the Commission on Radio Standards at the congresses of the last-named body at Washington, Brussels and Copenhagen. He had a considerable amount of administrative work to do in connection with these and his other official duties, and it is perhaps typical of his outlook that, after dealing for a few hours with such matters, he would suddenly break off, push the papers aside and remark that he really must go and do some work.

<div align="right">L. HARTSHORN</div>

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ERNEST HOWARD GRIFFITHS, D.Sc., F.R.S.

Principal E. H. Griffiths, who died on March 3, 1932, at the age of 80, was the last of a band of four workers who, collaborating together in the early 'nineties, did yeoman service in the development of the technique of high-temperature measurements. The names of Callendar, Griffiths, Heycock and Neville have now become household words in the extensive literature which has grown around the resistance thermometer. Griffiths's interest in the subject arose out of an enquiry from Heycock and Neville, who were searching for some better instrument for the measurement of the freezing points of alloys than the so-called "fixed zero" mercury thermometers available about 1890. Griffiths constructed a number of platinum resistance thermometers and proceeded to calibrate them by reference to fixed points. He was unable to reconcile his results with those of Callendar, who had just then published his difference formula. Griffiths had accepted the values of the fixed points given in the literature, one of which was the boiling point of sulphur as determined by Regnault. On the other hand, Callendar had based his formula on a direct comparison between the resistance of a spiral of platinum wire and the readings of an air thermometer.

To clear up the discrepancy they joined forces to redetermine the boiling point of sulphur, and showed that Regnault's value was about 4° too high. In this investigation they so admirably developed the technique of the sulphur determination that it has remained practically unchanged for the past 40 years. In addition they converted the resistance boxes of their day from crude appliances suitable for technical electrical measurements into instruments of precision.

At the same time Griffiths was working on the determination of the mechanical equivalent of heat. The primary object of this work was to check up the electrical standards which did not enjoy in those days the confidence we now have in their accuracy in absolute units. In his work on the mechanical equivalent of heat Griffiths had the assistance of Mr G. M. Clark.

Other investigations made in that period were the determination of the latent heat of evaporation of benzene with Miss Marshall, the influence of temperature on the specific heat of aniline, and the latent heat of evaporation of water; the last of these he regarded as one of his best pieces of work, and in later years he submitted it as a thesis for the D.Sc. degree of the University of Wales.

At Cambridge, Griffiths paid out of his own pocket for all the special apparatus he required. The writer recalls being told how a certain resistance bridge with a new design of plug contacts had cost £100 to build: an unheard-of sum for a bridge in those days. Griffiths protested to Horace Darwin who was the head of the Cambridge Instrument Company, but the latter produced the cost sheets and proved that the profit to the manufacturer was negligible.

For his work on the mechanical equivalent of heat Griffiths went to the extent of building a small laboratory in the grounds of his house. The design of his

equipment was not stinted on the grounds of the cost of the material: the writer once opened a piece of his apparatus and found that it contained two large platinum pots. Griffiths earned his money by coaching, and at that period a successful coach earned large fees. It involved intensive work during term time, so that scientific research had to be confined to vacations.

In 1902 he accepted the principalship of the University College, Cardiff. The change from the academic surroundings of Cambridge to the industrial surroundings of Cardiff was probably not to his liking. The problems of administration in a college of a democratic university were strange to him, and to one of his temperament must have proved irksome. And for some years he had not the solace of active experimental research owing to lack of facilities at that stage in the development of the college.

Griffiths devoted much energy to the furthering of the scheme for the building of a new college in Cathays Park. He was particularly interested in one of the buildings, the Viriamu Jones Memorial Research Laboratory. The details of this building were his special study and he insisted that it should be constructed entirely of non-magnetic materials.

In 1909 the writer was invited by Griffiths to work with him in the new Laboratory. Many months were spent in equipping the place, for all the jobs usually done by laboratory assistants were done by ourselves. There was no money for paying wages to a mechanic. Griffiths's hours of work were unconventional, but one could count upon his coming in during holiday periods and week-ends.

The investigation of the thermal capacities of metals from liquid-air temperatures up to 100° C. was the only scientific work he participated in at Cardiff.

In 1918 he retired to Cambridge and in that year his wife died. The last fourteen years of his life were somewhat lonely, for he had no children or near relatives. During the early part of this period of retirement he threw himself wholeheartedly into the work of the British Association, of which he became treasurer, but during the greater part of the last six years he was confined to his home with arthritis.

Throughout his life, and in the face of many bitter disappointments in connection with college work, he retained a spirit of optimism and youthfulness. One of the wishes he had "For any Boy" was:

> So let him live;
> Love work, love rest, love all that life can give:
> And when he feels too weary to feel joy
> Leave life with laughter—to some other boy.

His work received recognition by the award of the Hughes Gold Medal of the Royal Society: his election to an honorary fellowship of his old College, Sidney Sussex, and of Jesus College, Oxford.

The affection with which he was regarded was strikingly shown by the distinguished gathering in the chapel of Sidney Sussex College to pay the last tribute to an old colleague. The service closed with a hymn whose words were set to the old Welsh air "Ar hyd y nos" beloved by Griffiths.

EZER GRIFFITHS

H. CHAPMAN JONES, F.I.C.*

The death of Henry Chapman Jones on March 7, at the age of seventy-seven years, removed one who has held an honourable place in the teaching of chemistry and in the development of the science and practice of photography. Apart from an early association with Birkbeck College, Chapman Jones's career as a teacher of chemistry was bound up with the Royal College of Science, where he was successively assistant, demonstrator, and lecturer, from 1881 until 1914, when he retired. In the first few years of this long period he was on Sir Edward Frankland's staff, and later he served under T. E. Thorpe, W. A. Tilden, and H. B. Baker. For a number of years he presided over the North Laboratory in the old Royal College of Science buildings, and many who worked there will recall the quiet, serious, and kindly way in which he went about his duties, as well as the high standard of industry and accuracy which he expected of his students.

The main contributions to knowledge made by Chapman Jones were in the domain of photography, and were published principally under the auspices of the Royal Photographic Society. Following Sir William Abney in the application of scientific method to the problems of photography, Chapman Jones dealt with such matters as the densities of negatives, the factors affecting the sensitiveness of plates, media of high refractive power for photomicrography, and the relationship between the size of the particle and the colour of the image.

Besides original papers on these and other topics, Chapman Jones contributed many articles on photographic subjects to the technical press, while two books on photography came from his pen, namely, *Science and Practice in Photography*, which ran to four editions, and *Photography of To-day*, a popular treatise published in 1913. His associations with the Royal Photographic Society were very close, and after being honorary secretary from 1892 until 1897, and vice-president from 1897 until 1902, he became president of the Society in 1912. In addition, he was a Fellow of the Chemical and Physical † Societies and of the Institute of Chemistry.

Chapman Jones was a single-minded man, whose integrity and ideals were of the highest. His quiet, almost shy, old-world courtesy and his modesty were marks of an interesting personality which only those who knew him well could appreciate. At the same time, he held strong opinions on many subjects, and he did not hesitate to express these in his own direct, unimpassioned way. His gentle, warm-hearted nature revealed itself to best advantage in the circle of his own family and of those who knew him intimately. He is survived by a widow and two sons.

<div style="text-align: right">J. C. P.</div>

† Mr Jones was a Fellow of the Physical Society from November 27, 1880, until his death.

REVIEWS OF BOOKS

Revue d'Acoustique. First Number. Pp. 84 + xi. (Paris: Les Presses Universitaires de France, 49 Bd. St.-Michel, Paris.) 25 fr.

From the prospectus which accompanies this journal we learn that it is the aim of a panel of the leading French acousticians to provide for a long-felt want by publishing a review which shall report, co-ordinate, and tabulate information dealing solely with the science of sound. It is proposed that the review shall cover the whole domain of acoustics and contain purely theoretical papers and memoirs beside accounts of the most recent apparatus and methods obtaining in industrial practice.

To the present number M. Canac has contributed a vocabulary of proposed acoustical terms and definitions (based, to a large extent, on that already published in the *Journal of the Acoustical Society of America*); useful abstracts of some half-dozen papers; and an account of experimental work he has performed on the absorption of sound in tubes. Prof. Carrière, who is considering the absolute measurement of fundamental quantities in sound, has dealt in detail with the measurement of frequency, and proposes to continue by treating the measurement of amplitude, speed and pressure. In addition M. Perrin, writing on sound receivers, devotes most of his article to his own instruments and researches. A bibliographical review under nine headings and an eleven-page supplement consisting of a list of abbreviations of journal titles, completes the matter reported.

During the present year five numbers of the review will appear, while in succeeding years a subscription of 150 fr. will purchase the normal complement of six numbers. It is to be hoped that English subscribers will support the publication and thereby further their own interests as well as those of an important branch of physics.

E. J. I.

The Nature of a Gas, by Prof. LEONARD B. LOEB. Pp. x + 153. (London: Chapman and Hall.) 12s. 6d.

The Committee of the National Research Council of America appointed to deal with electrical insulation has interpreted its terms of reference widely and wisely, and has initiated a series of authoritative monographs on important fundamental properties of dielectric substances. Prof. Loeb's book is one of the first of these monographs and is intended to explain the nature and electric properties of gases to "students and engineers who are concerned with the problems of the behaviour of dielectrics as insulation."

Prof. Loeb is not afraid of elementary exposition. His book is divided into three parts, of which the first is an account of the Rutherford-Bohr atom. The second develops the kinetic picture of a gas and is, as one would expect from Prof. Loeb, a very clear and concise development of the fundamental principles of the kinetic theory. The third part, some forty pages long, is devoted to ionization phenomena, and the book concludes with a series of useful tables.

It is a curious mélange, but is eminently readable and very well fitted for its purpose. University students will find in it a very useful résumé of some of the fundamental properties of gases.

A. F.

A text-book of Thermodynamics, by F. E. Hoare, M.Sc., A.R.C.S., D.I.C. Pp. xii + 265. (London: E. Arnold and Co.) 15s.

There is room, in the literature of thermodynamics, for a volume which shall stress the number and the variety of the problems to which the principles of the subject may usefully be applied. Mr Hoare has set out to fill this gap, and has succeeded in producing a volume of much interest to workers. Three chapters of the book are devoted to a consideration of general principles, and the author then proceeds to treat of thermodynamic functions, the absolute scale of temperature, characteristic equations, change of state, problems of equilibrium, the Nernst heat theorem, thermodynamics of liquids and dilute solutions, applications of the principles of thermodynamics to electrical phenomena and to radiation and, finally, the variation of specific heat with temperature.

Thermodynamics is full of subtleties, and in a book which deals with applications rather than with fundamentals the discussion of some of the fundamental difficulties of the subject may incline to sketchiness. Thus, in the book before us, the treatment of the concept of temperature is a trifle naïve, and the handling of that very nice problem the increase of entropy in natural processes does not bring out the difficulties which encumber any adequate treatment of the subject.

These must not, however, be regarded as more than minor matters, and the author is to be congratulated on a volume which will be found very useful by university students of honours grade, and by research workers who look for clear and straightforward applications of the principles of thermodynamics to physical problems.

A. F.

Photoelectric Phenomena, by Prof. A. L. Hughes and Prof. L. A. DuBridge. Pp. xii + 531. (London: McGraw-Hill Publishing Co.) 30s.

Prof. Hughes's treatise on photoelectricity, published in 1913, and his *Report* of 1921, published in the *Bulletins* of the National Research Council, will be well known to most readers of these *Proceedings*. His colleague at St Louis and collaborator in the book under review, Prof. DuBridge, is also actively engaged in the investigation of photoelectric phenomena, and there is much evidence in the work of the close contact of the authors with the problems under discussion.

As would be expected, a great part of the book is taken up with the discussion of surface photoelectric effects, but there are also excellent accounts of volume effects in gases and vapours, of photoconductivity in solids and liquids and of photovoltaic effects. A chapter is devoted to the description of the effects obtained with X- and γ-rays, and there is a short account of the more important scientific and technical applications of photoelectric cells.

The book is directed primarily to physicists, and more particularly to experimental physicists. The sections dealing with experimental work are admirably written, the topics very judiciously selected and the emphasis very justly placed. These sections, and the excellent chapter on photoelectric technique, will be of use and interest both to pure physicists and to those who are more concerned with technical applications. The theoretical treatment is less detailed, and the bibliography somewhat less complete than on the experimental side. Nevertheless, a great many students of physics will find in chapter 6, on "Theories of photoelectric emission," a most useful and sufficiently detailed introduction to the theoretical aspects of photoelectricity.

A great deal of the subject matter of the book is here collected together for the first time, and in an eminently readable form, mainly from the viewpoint of the experimental

photoelectrician. The book fills a real gap in the literature of the subject, and it should be assured of a very warm welcome. There are a few minor slips and misprints—the most surprising oversight is in the section on γ-ray effects, where readers requiring further information are referred, not to the recent treatise of Rutherford, Chadwick and Ellis, but to two works published respectively in 1925 and 1928.

<div align="right">H. R. R.</div>

(1) *L'Idée générale de la Mécanique Ondulatoire et de ses premières applications*, by M. BOLL. Pp. 74. (Paris: Hermann et Cie.) 15 fr.

(2) *Exposé Électronique des Lois de l'Électricité*, by M. BOLL. Pp. 72. (Paris: Hermann et Cie.) 15 fr.

M. Marcel Boll is the author of an impressive and varied array of treatises and text-books, of which perhaps the best known in this country is the excellent (classical) *Introduction à la théorie des Quanta*, written in collaboration with C. Salomon.

In the first of the books now under review, M. Boll gives a brief general account of wave mechanics, suitable as an introduction to the more detailed and advanced treatises. The treatment is throughout formal and systematic—that is, there is no attempt to create for the reader, by means of models and analogies, an illusion of "understanding" the general outline of the new theories of matter. In other words, the book is elementary, rather than popular. It is clearly written, and can be recommended.

The second book will be of special interest to teachers of electricity, many of whom feel that electronic ideas and terminology should be introduced as early and as frequently as possible in even the most elementary lecture courses on the subject. It happens frequently, however, in practice—possibly through the exigencies of formal degree courses—that the electron is kept unduly in the background, and that an unnecessarily wide gap is left between the older and the more modern parts of electrical theory. For this reason, M. Boll's book should be of service to both teachers and students.

<div align="right">H. R. R.</div>

The Theory of Electric and Magnetic Susceptibilities, by J. H. VAN VLECK. Pp. xi + 384. (London: Oxford University Press.) 30s.

It is generally acknowledged that physicists who have devoted their attention to magnetic phenomena were greatly assisted by the publication of Debye's excellent article on magnetism in the *Handbuch der Radiologie*. In the opinion of the reviewer, Van Vleck's treatise on the theory of electric and magnetic susceptibilities is certain to be of similar assistance to all who are interested in polar molecules and the magnetic properties of substances. This treatise is no hastily assembled collection of facts and theories. It presents the clearest possible account of the present state of the mathematical theory of susceptibilities, an account which could only have been written by one who had read very widely and who had first-hand knowledge of much that he described.

In the first chapters, Van Vleck gives a very comprehensive review of the classical theories which form the basis of calculation of electrical susceptibilities, and he includes a much more adequate treatment of the Langevin-Debye formula than is to be found elsewhere. He discusses in detail its experimental verification, and, while not concerning himself with accounts of experimental technique, he presents a most valuable and stimulating survey of the phenomena exhibited by polar molecules.

His outline of the classical theory of magnetic susceptibility is masterly. The work of Miss Van Leeuwen, published in 1919, is so often passed over in silence, that it is most satisfactory to find that in this book it is given the prominence it deserves. Her proof of

the theorem that the calculated magnetic susceptibility by the application of classical Boltzmann mechanics to any dynamical system is equal to zero, is reproduced, and, in addition, Van Vleck provides an alternative proof of the theorem.

The author is at great pains to contrast the findings of the old quantum theory with those of the new. This does not represent a waste of space; for he shows, in a most lucid manner, and, truly, with a minimum of mathematics, how the inadequacies of the old quantum theory have been removed by the new, and how every calculation of magnetic susceptibility on the new mechanics has its analogue on classical theory.

As an introduction to the new mechanics, the chapter on quantum-mechanical foundations is likely to be most helpful to all interested in modern magnetism and allied phenomena. It pre-supposes no detailed knowledge of quantum mechanics, or of spectroscopic nomenclature or of the Schrödinger wave equation. The necessary perturbation theory and the theorems of spectroscopic stability are adequately presented, and the correlation and intermingling of wave functions and matrices which the author has successfully achieved make this introduction particularly valuable. Special attention is given to the general derivation of the Langevin-Debye formula and to a discussion of its limits of accuracy.

The treatment of magnetic susceptibility is not restricted to paramagnetic gases, but covers the salts of the rare earths and iron groups. Heisenberg's theory of ferromagnetism, together with its solution of the problem of the Weiss internal field, is fully discussed, and the experimental evidence which supports it is reviewed. The book ends with a brief treatment of optical phenomena, such as the Kerr effect, which are related to the problems discussed in the earlier chapters.

It is, then, a book which is bound to excite the serious attention of all who are interested in theoretical physics, and it is one which experimenters who measure dielectric constants or magnetic susceptibilities will be glad to possess; the author is much to be congratulated.

L. F. B.

Théorie de la Quantification dans la nouvelle Mécanique, by L. DE BROGLIE. Pp. xxviii + 250. (Paris: Hermann et Cie.) 70 fr.

This book may in some respects be regarded as a continuation of an earlier treatise of which an English translation, the *Introduction to the Study of Wave Mechanics*, appeared in 1930. After a brief historical introduction, it is divided into two parts, of which the first and shorter (on general principles and methods of quantization in wave mechanics) covers briefly, but clearly and adequately, the older ground. The second, and more important, part of the book deals with the general theory of quantization. As is usual in de Broglie's writings, the discussion is singularly lucid, and attention is very carefully focussed on fundamental principles. It scarcely needs to be said that the work is indispensable to all who are in any way interested in the wave aspects of the new mechanics.

Printing and paper are excellent, but, as with so many French books, reference would be facilitated by the provision of an alphabetical index and a paper-knife. There is, however, a very detailed table of contents.

H. R. R.

(1) *Sur une Forme plus restrictive des Relations d'Incertitude, d'après MM. Landau et Peierls,* by L. DE BROGLIE. Pp. 24. (Paris: Hermann et Cie.) 1932. 6 fr.

(2) *L'Existence du Neutron*, by I. CURIE and F. JOLIOT. Pp. 22. (Paris: Hermann et Cie.) 1932. 6 fr.

These "exposés de physique théorique," published under the editorship of L. de Broglie, form Nos. 31 and 32 of the general series of *Actualités Scientifiques et Industrielles*, of which the first thirty were spaced over the three years 1929–31. The plan of the series

is one which might well be copied in other countries—a similar series in English would certainly be profitable to students of physics, if not to the publisher.

De Broglie's pamphlet consists mainly of a commentary upon the recent paper of Landau and Peierls, "Erweiterung des Unbestimmtheitsprinzips für die relativistische Quantentheorie."* The subject-matter is of considerable theoretical importance, and of immediate interest in connection with nuclear dynamics.

The second "exposé" is even more obviously of topical interest. It deals with recent work on the disintegration of light nuclei, and more particularly with the researches carried out at the Curie Laboratory of the Paris Radium Institute.

<div align="right">H. R. R.</div>

Magnetic, Meteorological and Seismological Observations made at the Government Observatories, Bombay and Alibag in 1928, under the direction of Dr S. K. BANERJI. Pp. 160 with 5 Plates. (Calcutta, Government of India.) 22s. 6d.

This is the fiftieth volume of the series and is on the same lines as the previous volume† but has in addition an appendix on the climatology of Bombay. The rainfall was 85·7 in., 16·2 in. above the normal, and the mean temperature 79·9° F., 0·6° above the normal; mean wind 11·2 miles per hour, but on July 23 a gust of 54 miles per hour was recorded. The deviation of the compass had a mean value 3·8′ west as compared with 1·9′ west last year, and magnetic activity was generally slightly greater than last year.

For the 72 years ending 1920 the means for Bombay were: barometer 29·81 in., temperature 79·2° F., temperature in sunshine 139° F., humidity 75 per cent, wind 6·6· miles per hour, cloud covering 0·4 of the sky.

<div align="right">C. H. L.</div>

The Structure of Wind over Level Country. Meteorological Office Geophysical Memoirs No. 54, by the late M. A. GIBLETT, M.Sc., and other members of the staff. Pp. 119 with 21 Plates. (London: H.M. Stationery Office.) 10s.

This memoir, Dr Simpson tells us in his preface, is the outcome of a large amount of team work on the part of the staff of the Airship Division of the Meteorological Department, undertaken at the request of the Aeronautical Research Committee after the accident to Airship R 33 in 1925. The work was planned by the late Mr M. A. Giblett, who lost his life in the R 101 disaster, and Mr B. C. V. Oddie installed the necessary instruments at Cardington. The results of the six years' observations are embodied in a theory put forward by Mr C. S. Durst, according to which the large eddies, which produce gusts of wind beginning quickly and subsiding slowly, are due to convection currents set up in the air when the temperature near the ground is higher than that overhead. These currents are superposed on the general drift of the air, and take place in cells about a mile long in the direction of the wind, ¼ mile broad across the wind, and about the same height. The upward motion of the air in a cell takes place mainly near the back wall and the downward near the front wall. The air moving backwards near the ground is slowed down by viscosity and heated, while obstacles over which it passes produce the smaller eddies found in winds. The heated air ascending at the back of the cell expands and cools and its moisture-content condenses and forms clouds.

<div align="right">C. H. L.</div>

* *Zs. f. Phys.* **69**, 56 (1931).

† *Proc. Phys. Soc.* **44**, 112 (1932). The deviation of the compass and its increase there given were in minutes, not degrees.

Recent Advances in Physical Chemistry, by SAMUEL GLASSTONE, D.Sc. Pp. viii + 470. (London: J. and A. Churchill.) 15*s.*

There is to-day considerable overlap in the sciences, and physical chemistry takes to itself results in certain branches of physical science—molecular spectra and the theory of surface forces are cases in point—in which the knowledge of the physicist may reasonably be expected to be in advance of, and more critical than, that of the physical chemist.

But it is simply impossible to keep a critical and detailed knowledge of all those branches of physical chemistry which may be supposed to be of special interest to the physicist, and those of us who have to deal with borderline subjects owe a debt of gratitude to anyone who, having the requisite knowledge, is disinterested enough to provide us with accurate and critical summaries of those branches of the subject which incline more to chemistry than to physics.

The different sections of Dr Glasstone's book deal with valency theories, the parachor, dipole moments, molecular spectra, homogeneous gas reactions, photochemical reactions, the properties of surfaces, heterogeneous catalysis, solubility, strong electrolytes, and acid-base catalysis. The author has the teacher's flair for exposition, and handles his subjects in clear, scholarly and adequate fashion. He does not fall into the error of assuming that his readers are likely to possess an expert's knowledge of the subject.

The book can be strongly recommended to those who desire a brief conspectus of some of the more important advances in physical chemistry during the past decade.

Not the least valuable feature of the book is the bibliographical appendix to each chapter. The entries therein amount in the aggregate to nearly five hundred items.

A. F.

Hearing in Man and Animals, by R. T. BEATTY, M.A., B.E., D.Sc. Pp. xi + 223. (London: G. Bell and Sons, Ltd.) 12*s.*

The purpose of this book as set forth by the author in his preface is to co-ordinate work on audition in the different fields of research—physics, anatomy, physiology, etc. He rightly points out that there is a tendency, in this as in other branches of biophysics, for a worker to keep to the path indicated by the usual scope of research in his own branch of science, and to write in the *argot* peculiar to that branch, ignoring papers in the other branches or perhaps not understanding the technical terms affected by his colleagues on the other side. This contention of the author is borne out to a certain extent by the Physical Society's *Discussion on Audition* in which the contributions can be grouped, as to terminology, in three sections—physical, biological and psychological. At any rate the author has ably achieved his purpose in the treatise under review. Simple language has been used; nothing more is demanded of the reader than a lively interest and a certain scientific grounding. The bibliography at the end of each chapter is not extensive, but references to text-books where further information is available are given. Unlike many text-books, this work enlivens the course of the reader by anecdotes, historical curiosities and imaginative pictures of bygone stages in scientific investigation. Nevertheless it is quite up-to-date, and not its least valuable feature is the insight it gives to the scientific investigator into lacunae in research, to which he might with profit direct his attention. Altogether, a very readable book, and one which should be added to every bookshelf which possesses the *Discussion on Audition*.

E. G. R.

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PROCEEDINGS AT THE MEETINGS
OF THE PHYSICAL SOCIETY

SESSION 1931–32

Except where the contrary is stated, the meetings were held at the Imperial College of Science and Technology, South Kensington.

October 16, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Sydney Bennett Fulford was elected to Fellowship of the Physical Society. The President announced that Charles Norman Smyth, Denis Taylor, Nelson Laycock, Eric Graham Knowles, Ronald Osmond Jenkins, Maurice Hancock, E. H. Copsey, J. R. Tillman, Alan E. W. Austen, George Ronald Cooper, Eric Henry Dock, Charles F. Brockelsby, Frederick C. Le Manquais and Alec A. Fletcher had been admitted to Student Membership of the Society.

2. The following papers were read:

"Young's modulus for two directions in a steel bar," by G. A. WEDGWOOD, M.Sc.

"The effective mass of flexible discs and conical diaphrams used for sound-reproduction," by N. W. McLACHLAN, D.Sc., M.I.E.E., F.Inst.P.

"Further study of diffusion for the infinite plane sheet," by A. T. McKAY, M.Sc.

3. A demonstration of the effects of mechanical disturbance on a neon lamp was given by Miss TERESA J. DILLON, M.Sc., and Miss C. M. LOVETT, B.Sc.

November 6, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Maurice K. Taylor, Alexander Charles George Menzies, Jack Ernest Rayner Constable and P. B. Moon were elected to Fellowship of the Physical Society.

The President announced that the Council had elected A. H. M. Radwan, W. F. Hilton, A. G. Gaydon and Chetpat Ramaswami Sundaram to Student Membership of the Society.

2. The presidential address, entitled "The Expanding Universe," was delivered by Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S.

A vote of thanks to the President was proposed by Dr W. H. Eccles, seconded by Mr C. C. Paterson, and carried by acclamation.

November 20, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. The following papers were read:

"The band spectrum of zirconium oxide," by Miss F. LOWATER, Ph.D., F.R.A.S.

"Lattice-distortion of cold-drawn constantan wire," by W. A. WOOD, M.Sc., Physics Department, National Physical Laboratory.

"A remote electrically-recording accelerometer with particular reference to wheel-impact measurements," by F. AUGHTIE, Ph.D., M.Sc., Engineering Department, National Physical Laboratory.

"A remote electrically-recording load-gauge for wheel-impact measurements," by F. AUGHTIE, Ph.D., M.Sc., Engineering Department, National Physical Laboratory.

"Wireless echoes of short delay," by E. V. APPLETON, F.R.S., and G. BUILDER, B.Sc.

2. Demonstrations of (*a*) A contrivance for demonstrating the law of errors; (*b*) A new type of surface-tension-meter; and (*c*) A new type of static electrometer were given by Prof. KERR GRANT, M.Sc.

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December 4, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Bernard Rhodes, Leslie H. Daniel and Donald Hewitt were elected to Fellowship of the Physical Society.

2. The following papers were read:

"Some properties of the sound emitted by airscrews," by C. F. B. KEMP, A.R.C.S., B.Sc., D.I.C.

"The latent heat of some refrigerants," by EZER GRIFFITHS, D.Sc., F.R.S., and J. H. AWBERY, B.A., B.Sc., F.Inst.P., Physics Department, National Physical Laboratory.

"The specific volumes of some gaseous refrigerants," by J. H. AWBERY, B.A., B.Sc., F.Inst.P., and EZER GRIFFITHS, D.Sc., F.R.S., Physics Department, National Physical Laboratory.

"The basic law of the wet-and-dry-bulb hygrometer at temperatures from 40° to 100° C.," by J. H. AWBERY, B.A., B.Sc., F.Inst.P., and EZER GRIFFITHS, D.Sc., F.R.S., Physics Department, National Physical Laboratory.

"The water-content of saturated air at temperatures up to 100° C.," by J. H. AWBERY, B.A., B.Sc., F.Inst.P., Physics Department, National Physical Laboratory.

3. A demonstration of the effects produced when slow-velocity liquid jets fall on a barrier was given by J. H. BRINKWORTH, D.Sc., A.R.C.S., D.I.C.

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January 5, 6 and 7, 1932.

The twenty-second Annual Exhibition of the Physical and Optical Societies was held at the Imperial College of Science.

Discourses were delivered as follows:

January 5. "Photocells: the valves which operate by light," by C. C. PATERSON, O.B.E., M.I.E.E., F.Inst.P.

January 6. "Photographic shutters and their properties," by T. SMITH, M.A., F.Inst.P.

January 7. "Reminiscences," by Sir OLIVER LODGE, D.Sc., LL.D., F.R.S., F.Inst.P.

January 15, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Frank Aughtie and A. Stanley M. Symons were elected to Fellowship of the Physical Society.

The President announced that John Spencer Roebuck, George Ernest Tunnicliffe and Arthur Cecil Challands had been admitted to Student Membership of the Society.

2. The following papers were read:

"Some thermomagnetic effects in nickel and iron," by SHIH-CHEN T'AO, M.S. (Yenching), and WILLIAM BAND, M.Sc., Yenching University, Peking.

"The electroscope capacity balance," by E. S. BROWN, D.Sc., A.C.G.I., University of Melbourne.

"On periodic movements of the negative glow in discharge tubes," by Miss W. A. LEYSHON, Ph.D.

"A cathode-ray oscillographic method of measuring inductance," by G. I. FINCH, M.B.E., and R. W. SUTTON, Imperial College of Science and Technology.

"The measurement of electrical resistance in terms of a mutual inductance and a period," by H. R. NETTLETON, D.Sc., and F. H. LLEWELLYN, B.Sc., Birkbeck College.

February 5, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Eric Norton Grindley, Cecil George Lemon and Morris E. Sions were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The collisional friction experienced by vibrating electrons in ionized air," by E. V. APPLETON, F.R.S., and F. W. CHAPMAN, M.Sc.

"Surface heating by neutralized positive rays before and after return to normal state," by M. C. JOHNSON, M.A., D.Sc., Birmingham University.

"The rapid determination of the moisture-content of seeds and other granular substances," by R. M. DAVIES, M.Sc., University College of Wales, Aberystwyth.

"Electrolytic water-transport and ionic transport numbers," by H. C. HEPBURN, Ph.D., Birkbeck College.

"On the magnetic susceptibilities of some nickel compounds," by R. A. FEREDAY, B.Sc., East London College.

February 19, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Arthur Osborne Jones and W. Grenville Symons were elected to Fellowship of the Physical Society.

2. The following papers were read:

"Electrons and light quanta," by Sir AMBROSE FLEMING, F.R.S., Emeritus Professor of Electrical Engineering in the University of London.

"The spherical shell method of determining the thermal conductivity of a thermal insulator," by S. E. GREEN.

"The optics of photometric measurements," by T. SMITH, M.A., National Physical Laboratory.

"Sir A. S. Eddington's recent theories," by W. N. BOND, M.A., D.Sc., F.Inst.P., Lecturer in Physics in the University of Reading.

"Some principles governing the design of Kerr cells," by W. D. WRIGHT, Ph.D., A.R.C.S., D.I.C.

"On the forces acting on drops in an electric field," by G. D. WEST, D.Sc., F.Inst.P.

March 4, 1932.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. D. Narayanamurti was elected to Fellowship of the Physical Society.

2. The following papers were read:

"A vacuum calorimeter for high temperatures," by L. G. CARPENTER, B.A., B.Sc., and T. F. HARLE, B.Sc., University College, Southampton.

"A new tilted electrometer," by Hugh Carmichael, B.Sc., University of Edinburgh.

"On the symmetrical modes of vibration of truncated conical shells; with applications to loud-speaker diaphragms," by N. W. McLachlan, D.Sc., M.I.E.E.

3. A demonstration on some lecture experiments in alternating currents was given by W. Bennett, A.R.C.S., B.Sc.

March 18, 1932.

Annual general meeting.

Prof. Sir Arthur Eddington, M.A., D.Sc., F.R.S., in the Chair.

1. The Minutes of the preceding Annual Meeting were read and confirmed.

2. The Reports of the Council and of the Hon. Treasurer were presented and adopted.

3. The following Officers and Members of Council were elected for the year 1932–1933:

President: Prof. A. O. Rankine, O.B.E., D.Sc., F.Inst.P.

Vice-Presidents (who have filled the office of President): Sir Oliver J. Lodge, D.Sc., LL.D., F.Inst.P., F.R.S.; Sir Richard Glazebrook, K.C.B., Sc.D., F.Inst.P., F.R.S.; Sir Arthur Schuster, Ph.D., Sc.D., F.Inst.P., F.R.S.; Sir J. J. Thomson, O.M., Sc.D., F.Inst.P., F.R.S.; Prof. C. Vernon Boys, F.Inst.P., F.R.S.; Prof. C. H. Lees, D.Sc., F.Inst.P., F.R.S.; Prof. Sir W. H. Bragg, K.B.E., M.A., F.Inst.P., F.R.S.; Alexander Russell, M.A., D.Sc., F.Inst.P., F.R.S.; Sir F. E. Smith, K.C.B., D.Sc., F.Inst.P., F.R.S.; Prof. O. W. Richardson, M.A., D.Sc., F.R.S.; W. H. Eccles, D.Sc., F.Inst.P., F.R.S.; Prof. Sir A. S. Eddington, M.A., D.Sc., F.R.S.

Vice-Presidents: A. B. Wood, D.Sc., F.Inst.P.; T. Smith, M.A., F.Inst.P., F.R.S.; J. S. G. Thomas, D.Sc.; J. Guild, A.R.C.S., D.I.C., F.Inst.P.

Hon. Secretaries: Allan Ferguson, M.A., D.Sc., F.Inst.P.; Ezer Griffiths, D.Sc., F.Inst.P., F.R.S.

Hon. Foreign Secretary: Prof. O. W. Richardson, M.A., D.Sc., F.R.S.

Hon. Treasurer: R. S. Whipple, M.I.E.E., F.Inst.P.

Hon. Librarian: J. H. Brinkworth, M.Sc., A.R.C.S.

Ordinary Members of Council: Lewis Simons, D.Sc., F.Inst.P.; J. H. Awbery, B.Sc., F.Inst.P.; Prof. J. A. Crowther, M.A., Sc.D., F.Inst.P.; W. Jevons, D.Sc., D.I.C., F.Inst.P.; Prof. W. Wilson, Ph.D., D.Sc., F.R.S.; D. Owen, B.A., D.Sc., F.Inst.P.; Miss M. O. Saltmarsh, Ph.D.; Major I. O. Griffith, M.A.; Prof. H. R. Robinson, F.Inst.P., F.R.S.; Prof. G. F. J. Temple, Ph.D., D.Sc.

4. The Duddell Medal, 1931, was presented to Prof. C. T. R. Wilson, F.R.S.

5. The following votes of thanks were moved and were carried by acclamation: To the retiring Officers and Council, proposed by Mr R. W. Paul and seconded by Prof. G. Temple; to the Honorary Auditors, proposed by Dr J. S. G. Thomas and seconded by Mr T. Smith; and to the Governors of the Imperial College of Science, proposed by Prof. W. Wilson and seconded by Mr F. J. W. Whipple.

Ordinary meeting following the annual general meeting.

Prof. A. O. Rankine, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. James Garton Bower, Denis Edward Alan Jones and William George Pye were elected to Fellowship of the Physical Society.

The President announced that D. G. Jackess had been admitted to Student Membership of the Society.

2. The following papers were read:

"The first spark spectrum of arsenic (As II)," by A. S. Rao, M.A., M.Sc., Solar Physics Observatory, Kodaikanal, S. India.

"The photographic measurement of the absorption coefficients of gamma-rays from radium (B + C)," by J. S. Rogers, B.A., M.Sc., F.Inst.P., Senior Lecturer in Natural Philosophy, University of Melbourne.

"The derivation of Maxwell's equations from the equations of the quantum theory," by M. Fahmy, The Egyptian University, Cairo.

April 15, 1932.

Prof. A. O. Rankine, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. H. Levy, Herbert Frank Winny, Edward Burke, Stanley John Welton, Charles Henry Kemp and A. S. Radford were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The measurement of reflection coefficients for oblique incidence," by H. E. Beckett, B.Sc., Building Research Station.

"Studies in Interferometry. (1) A new type of interference refractometer," by W. Ewart Williams, M.Sc., Lecturer in Physics, King's College, London.

"A direct-reading γ-ray electroscope," by L. G. Grimmett, B.Sc., Assistant Physicist, Westminster Hospital, London.

May 6, 1932.

Meeting held in the Physiological Laboratory of University College, Gower Street, W.C. 1.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. Cyril James Beesley, Edward William Herbert Selwyn and Leonard Ernest Sharp were elected to Fellowship of the Physical Society.

2. A series of demonstrations, including the following, was given:

The heat-production of nerve.
Galvanometer-amplification by photoelectric cell.
Photoelectric cell used as micrometer and for other purposes.
Vapour-pressure measurement by thermopile.
Muscle-heat production.
Thermoelastic phenomena.

3. A lecture was delivered by Prof. A. V. HILL, F.R.S., on "The measurement and analysis of the heat-production of nerve."*

May 20, 1932.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. Donald Walter Carter, Walter Wilson, Arnold Joseph Holland, John Guyscliffe Holmes, Brian Clifford Fleming-Williams, Ronald James Cox, G. G. Blake and Henry L. Brose were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The fall of potential in a charged insulated cable," by D. K. McCLEERY, B.Sc., A.M.I.E.E.

"On the representation and calculation of the results of gravity surveys with torsion balances," by Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., Professor of Physics in the Imperial College of Science and Technology.

"Some observations with a gravity-gradiometer," by Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., Professor of Physics in the Imperial College of Science and Technology.

"The propagation along the earth of radio waves on a wave-length of 1·6 metres," by R. L. SMITH-ROSE, D.Sc., Ph.D., A.M.I.E.E., and J. S. McPETRIE, B.Sc., A.M.I.E.E.

"Notes on surface-tension measurement," by ALLAN FERGUSON, M.A., D.Sc., F.Inst.P., and S. J. KENNEDY, B.Sc.

3. A demonstration of an experiment on capillary adsorption was given by D. OWEN, B.A., D.Sc., F.Inst.P.

* The substance of this lecture will be found in *Chemical Wave Transmission in Nerve* (Cambridge University Press, 1932).

June 3, 1932.

Prof. A. O. Rankine, O.B.E., D.Sc., F.Inst.P., and subsequently
Allan Ferguson, M.A., D.Sc., in the Chair.

A discussion on vision* was held, the following papers being read and discussed:

J. Guild, "Some problems of visual reception." F. Allen, "The trichromatic theory and its explanatory power." J. Drever, "The fundamental colours and the colour triangle." J. Guild, "The interpretation of quantitative data in visual problems." H. Hartridge, "Colour vision by modified white light." S. Hecht, "A quantitative formulation of colour vision." L. C. Martin, F. L. Warburton and W. J. Morgan, "Some recent experiments on the sensitiveness of the eye to differences in the saturation of colours." W. Peddie, "The essence and present position of the trichromatic theory." L. F. Richardson, "The measurability of sensations of hue brightness or saturation." W. D. Wright, "The significance of colour fatigue measurements." A. Bruckner, "Untersuchungen zur Dunkel-adaptation des menschlichen Auges." R. A. Houstoun, "New observations on the Weber-Fechner law." K. Koffka, "A new theory of brightness constancy; a contribution to a general theory of vision." D. Roaf, "The sensation of light as a photochemical process." W. S. Stiles and B. H. Crawford, "Equivalent adaptation levels in localised retinal areas." T. Smith, "The colour triangle and colour discrimination." H. Banister, "Retinal action time." F. C. Bartlett, "A note on the visual perception of depth." C. E. Ferree and G. Rand, "A study of the refractive conditions for the peripheral field of vision." R. A. Granit, "The physiological significance of the retinal synapses." J. Parsons, "Some biological aspects of visual measurements." H. Pieron, "Les lois du temps du chroma." A. von Pflugk, "Die Lehre von der Akkommodation im Lichte der neueren Forschung." R. S. Creed, "Visual acuity and retinal structure." E. F. Fincham, "The mechanism of accommodation and the recession of the near point." H. Hartridge, "Visual acuity and the resolving power of the eye." H. E. Roaf, "Some experimental observations on the properties of the receiving organs in the retina." F. Roessler, "A subjective examination of eye astigmatism."

June 11, 1932.

Meeting held at University College, Nottingham.

Prof. A. O. Rankine, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. The following papers were read:

"Relation of electron collisions in gases to the Raman spectra," by H. L. Brose.

"Electrodeless discharges," by J. S. Townsend.

* Published in a separate volume.

"Intensity measurements in the spectra of electrodeless discharges," by J. E. KEYSTON.

"Directional investigation of atmospherics," by L. G. H. HUXLEY.

"Raman effect in electrolytes," by L. A. WOODWARD.

"Limiting effective cross-section of gas atoms with respect to very slow electrons," by E. H. SAAYMAN.

2. Some demonstrations were given under the supervision of N. DAVY, H. M. BROWNING and A. H. FRANKS.

June 17, 1932.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

The Seventeenth Guthrie Lecture was delivered by Prof. MAX PLANCK, on "The concept of causality."

July 8, 1932.

Special general meeting.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

The following resolutions were passed:

1. That the name of the Society be changed to "The Physical Society."

2. That, subject to the proposed change of name of the Society being approved by the Board of Trade and a new certificate of incorporation of the Society in the name of "The Physical Society" being issued by the Registrar, the regulations contained in the document submitted to the meeting and for the purpose of identification signed by the chairman thereof be approved and adopted as the articles of association of the Society in substitution for and to the exclusion of all existing articles thereof.

Ordinary meeting following the special general meeting.

Prof. A. O. RANKINE, O.B.E., D.Sc., F.Inst.P., in the Chair.

1. Walter Edgar Flood, James Lewis Pugh Macnair and Lionel V. Cole were elected to Fellowship of the Physical Society.

2. The following papers were read:

"The determination of refractivity temperature coefficients for liquids," by JOHN J. MANLEY, M.A., D.Sc., Oxon., Fellow of Magdalen College, Oxford.

"The axial sound-pressure due to diaphragms with nodal lines," by N. W. McLACHLAN, D.Sc., M.I.E.E.

"The accession to inertia of flexible discs vibrating in a fluid," by N. W. McLachlan, D.Sc., M.I.E.E.

"A method for deducing accurate values of the lattice spacing from X-ray powder photographs taken by the Debye-Scherrer method," by A. J. Bradley, Ph.D., and A. H. Jay, M.Sc.

"Ionization charts of the upper atmosphere," by G. Millington, M.A., B.Sc.

"Further investigation of the arc spectrum of arsenic" by A. S. Rao, M.A., M.Sc., Solar Physics Observatory, Kodaikanal, India.

REPORT OF COUNCIL FOR THE PERIOD ENDING
FEBRUARY 29, 1932

MEETINGS

DURING the period covered by the Report 9 Ordinary Science Meetings were held at the Imperial College of Science. At these meetings 40 papers were presented and 10 demonstrations given.

On June 19, 1931, a Discussion on Audition was held, to which 18 papers were contributed. On June 20, 1931, members of the Society and their friends visited Reading University at the invitation of Professor Crowther; at this meeting three papers were read and 17 demonstrations were given by the staff of the Physics Department.

A lecture entitled "Cohesion" was delivered on May 1, 1931, by Professor J. E. Lennard-Jones, D.Sc., Ph.D.

The Presidential Address entitled "The Expanding Universe" was delivered on November 6, 1931, by Sir Arthur Eddington, M.A., D.Sc., F.R.S.

EXHIBITION

The Twenty-Second Annual Exhibition arranged jointly by the Physical and Optical Societies was held on January 5, 6 and 7, 1932, at the Imperial College of Science, by courtesy of the Governing Body. The Research and Experimental Section included exhibits from 31 sources. Trade exhibits were shown by 82 firms. Discourses were given by C. C. Paterson, O.B.E., M.I.E.E., F.Inst.P., on "Photocells: the Valves which operate by Light"; by T. Smith, M.A., F.Inst.P., on "Photographic Shutters and Their Properties"; and by Sir Oliver Lodge, D.Sc., LL.D., F.R.S., F.Inst.P., on "Reminiscences."

REPRESENTATIVES ON OTHER BODIES

Dr D. Owen and Dr Ezer Griffiths have been appointed representatives on the Board of the Institute of Physics; and Sir William Bragg, Professor E. A. Owen and Professor F. L. Hopwood have been reappointed representatives of the Society on the British X-Ray Unit Committee.

THE DUDDELL MEDAL

At the Annual Meeting on March 20 the Eighth (1930) Duddell Medal was presented to Professor Sir Ambrose Fleming, F.R.S. The Council has awarded the Ninth (1931) Duddell Medal to Professor C. T. R. Wilson, F.R.S.

GUTHRIE LECTURE

Sir Richard Glazebrook, K.C.B., M.A., Sc.D., F.R.S., delivered the Sixteenth Guthrie Lecture on May 15, 1931, the subject being "Standards of Measurement, their History and Development."

SPECIAL REPORT

A report on "Band Spectra" has been received from W. Jevons, D.Sc., Ph.D., F.Inst.P., in accordance with the request of the Council, and is now passing through the press.

OBITUARY

The Council records with deep regret the deaths of two Honorary Fellows, Professor A. A. Michelson and Dr S. W. Stratton; and of the following Fellows: Emeritus Professor Archibald Barr, Dr D. W. Dye, Mr Channell Law, Colonel E. D. Malcolm, Professor J. Sampson and Mr H. Tomlinson.

MEMBERSHIP ROLL

The number of Honorary Fellows on the Roll on December 31, 1931, was 9. At the same date Ordinary Fellows numbered 773, and Students 60.

The changes in the membership of the Society are shown in the appended table:

	Total Dec. 31, 1930	Changes during 1931		Total Dec. 31, 1931
Honorary Fellows	11	Deceased	2	9
Ordinary Fellows	754	Elected	26	
		Student transfers	18	
			44	
		Deceased 5		
		Resigned or lapsed 20		
			25	
		Net increase	19	773
Students ...	51.	Elected	29	
		Trans. to Fellow ... 18		
		Resigned 2		
			20	
		Net increase	9	60
Total Membership	816	Net increase	26	842

REPORT OF THE HONORARY TREASURER

THE accounts for the year ended December 31, 1931, show a balance of £55. 15s. 7d.

The reduction in the amount charged by the Institution of Electrical Engineers to the Society for "Science Abstracts" is due to the increased annual grant made by the American Physical Society.

The cost of the report of the Discussion on Audition amounted to £294. 4s. 2d.; it is anticipated that part of this expenditure will be recouped by the sale of the report.

£900 of $2\frac{1}{2}$ % Consolidated Stock was purchased at a cost of £539. 12s. od.

The Society's investments have been valued at market prices on December 31, 1931, through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank.

<div style="text-align: right;">

(Signed) ROBERT S. WHIPPLE

Honorary Treasurer

</div>

March 9, 1932

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31st DECEMBER, 1931

EXPENDITURE

1930 £ s. d.	EXPENDITURE	£ s. d.	£ s. d.
600 0 0	To Stock of Publications at 1st January, 1931		779 0 0
	Science Abstracts	474 0 0	
	Less Rebate for 1930	235 0 0	
566 0 0			239 0 0
	Ordinary Publications:		
1374 12 10	"Proceedings"	1315 2 8	
80 7 7	"Bulletin" and Notices	78 13 2	
72 19 6	General	72 14 11	1466 10 9
116 8 5	Postage on Publications		113 15 2
	Printing and Publishing Discussion on "Audition"		294 4 2
66 4 7	Reporting at Meetings		65 1 10
33 11 2	Expenses at Meetings		37 9 1
744 10 2	Exhibition Expenses	824 5 11	
52 17 9	Optical Society Exhibition Account	67 6 0	891 11 11
5 19 2	Periodicals and Library		6 7 0
	Administration Expenses:		
169 11 8	Institute of Physics	170 4 4	
112 18 9	Office Expenses	105 15 5	275 19 9
20 0 0	Guthrie Lecture (Honorarium)		20 0 0
378 10 2	Share of cost of printing and publishing "Photo-Electric Cells and their Applications"		
36 0 0	Printing List of Members		
	Balance, being excess of Income over Expenditure, carried forward		55 15 7
£4430 11 9			£4244 15 3
	To Balance carried to Accumulated Fund		£55 15 7

INCOME

1930 £ s. d.	INCOME	£ s. d.	£ s. d.
779 0 0	By Stock of Publications at 31st December, 1931		747 0 0
	Subscriptions:		
1242 1 3	Fellows*	1327 2 2	
8 3 0	Students (Voluntary†)	7 7 0	
28 17 6	For "Science Abstracts" and Advance	18 7 6	
102 4 0	Proofs	102 17 0	1455 13 8
819 17 8	Sale of Publications	833 14 11	
	Less Subscriptions received in advance	207 3 2	626 11 9
165 5 4	Advertisements in "Proceedings"		194 5 10
257 10 1	Dividends from Investments and Bank Interest	249 10 6	
	Add: Income Tax refunded Jan.–April 4, 1931	15 1 11	
	Income Tax claimed April 5–Dec. 31, 1931	17 6 0	
		281 18 5	
	Less: Transferred to Duddell Memorial Fund	20 0 0	
		261 18 5	
	Add: Dividends due at December 31, 1931	35 10 9	
	Income Tax claimed	11 9 10	308 19 0
805 10 1	Exhibitors' Payments on account of Exhibition		
200 0 0	Royal Society Grant for Publications		
22 2 10	Balance, being excess of Expenditure over Income, carried forward		912 5 0
£4430 11 9			£4244 15 3
	By Balance brought forward		£55 15 7

* Eighty-eight Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total rebate being £31. 4s. 11d.

† Voluntary subscriptions are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT 31ST DECEMBER, 1931

LIABILITIES

	£ s. d.	£ s. d.
Accumulated Fund:		
As per last Balance Sheet	1895 14 11	
Entrance Fees, 1931	38 17 0	
Balance brought forward from Income and Expenditure Account	55 15 7	
	1990 7 6	
Less Decreased value of Investments	259 12 0	1730 15 6
Life Compositions:		
As per last Balance Sheet		2384 0 0
Duddell Memorial Trust Fund:		
As per last Balance Sheet	434 12 9	
Less Decreased value of Investments	24 0 0	410 12 9
W. F. Stanley Trust Fund (for the "Bulletin"):		
As per last Balance Sheet	265 0 0	
Less Decreased value of Investments	110 0 0	155 0 0
A. W. Scott Bequest		250 0 0
Sundry Creditors		1143 4 11
Subscriptions paid in advance:		
Members	21 15 10	
Publications	207 3 2	228 19 0
		£6302 12 2

ASSETS

	£ s. d.	£ s. d.
Investments at Market Value on December 31, 1931:		
£1500 Consolidated Stock 2½%	825 0 0	
£1000 War Loan 5% 1929/47 Inscribed "A" Account	960 0 0	
£400 War Loan 5% 1929/47 Inscribed "B" Account	384 0 0	
£650 Funding Loan 4% 1960/90	539 0 0	
£500 India 3½% Stock	250 0 0	
£254. 2s. 9d. New South Wales 5% Stock 1935/55	182 0 0	
£211 London County Consolidated 4½% Stock	186 0 0	
£400 Lancaster Corporation 3% Redeemable Stock	268 0 0	
£399 London Midland and Scottish Railway 4% Debenture Stock	275 0 0	
£1000 London Midland and Scottish Railway 4% Preference Stock	460 0 0	
£500 London and North Eastern Railway 4% Debenture Stock	325 0 0	
£150 Southern Railway 5% Debenture Stock	128 0 0	
£300 Southern Railway Preferred Ordinary Stock	111 0 0	
£442 Southern Railway Deferred Ordinary Stock	44 0 0	4937 0 0
Dividends due from Investments		35 10 9
Stock of Publications (as per Treasurer's valuation)		747 0 0
Subscriptions due		22 1 0
Sundry Debtors		279 12 5
Inland Revenue—Income Tax reclaimed for 1931		28 15 10
Cash at Bank:		
On Deposit Account	100 0 0	
On Current Account	134 8 5	
Cash in hand	18 3 9	252 12 2
		£6302 12 2

ROBERT S. WHIPPLE, *Honorary Treasurer.*

We have audited the above Balance Sheet and have obtained all the information and explanations we have required. We have verified the Bank Balances and the Investments. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the books of the Society.

SPENCER HOUSE, SOUTH PLACE, E.C. 2
9th March 1932.

KNOX, CROPPER & CO.,
Chartered Accountants
(Signed) F. H. SCHOFIELD.
(Signed) W. S. TUCKER.

Examined and approved on behalf of the Society

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1931

	£	s.	d.
128 Fellows paid £10	1280	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20	20	0	0
1 Fellow paid £20. 10s.	20	10	0
14 Fellows paid £21	294	0	0
23 Fellows paid £31. 10s.	724	10	0
	£2384	0	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN")

	£	s.	d.		£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	111	0	0	Carried to Balance Sheet . .	155	0	0
£442 Southern Railway Deferred Ordinary Stock	44	0	0				
	£155	0	0		£155	0	0

DUDDELL MEMORIAL TRUST FUND

CAPITAL

	£	s.	d.		£	s.	d.
£400 War Loan 5 % 1929/47 Inscribed Stock .	384	0	0	Carried to Balance Sheet . .	384	0	0

REVENUE

	£	s.	d.		£	s.	d.
Balance at 31st December, 1930 . . .	26	12	9	Honorarium to Medallist . . .	20	0	0
Dividends	20	0	0	Balance carried to Balance Sheet .	26	12	9
	£46	12	9		£46	12	9